CHAPTER 27:
THE THEORY OF ACTIVE PORTFOLIO MANAGEMENT

1. a. Define $R = r - r_f$

   Note that we compute the estimates of standard deviation using 4 degrees of freedom (i.e., we divide the sum of the squared deviations from the mean by 4 despite the fact that we have 5 observations), since deviations are taken from the sample mean, not the theoretical population mean.

   $E(R_B) = 11.16\%$  $\sigma_B = 21.24\%$

   $E(R_U) = 8.42\%$  $\sigma_U = 14.85\%$

   Risk neutral investors would prefer the Bull Fund because its performance suggests a higher mean.

   b. Using the reward-to-volatility (Sharpe) measure:

   $S_B = \frac{E(r_B) - r_f}{\sigma_B} = \frac{E(R_B)}{\sigma_B} = \frac{11.16}{21.24} = 0.5254$

   $S_U = \frac{E(r_U) - r_f}{\sigma_U} = \frac{E(R_U)}{\sigma_U} = \frac{8.42}{14.85} = 0.5670$

   The data suggest that the Unicorn Fund dominates for a risk averse investor.

   c. The decision rule for the proportion to be invested in the risky asset is given by the following formula:

   $y = \frac{E(r) - r_f}{\sigma} = \frac{E(R)}{0.01A\sigma^2}$

   This value of $y$ maximizes a mean-variance utility function of the form:

   $U = E(r) - 0.005A\sigma^2$

   For utility functions of this form, Sharpe’s measure is the appropriate criterion for the selection of optimal risky portfolios. An investor with $A = 3$ would invest the following fraction in Unicorn:

   $y_U = \frac{8.42}{0.01 \times 3 \times 14.85^2} = 1.2727$

   Note that the investor seeks to borrow in order to invest in Unicorn. In that case, his portfolio risk premium and standard deviation would be:

   $E(r_P) - r_f = 1.2727 \times 8.42\% = 10.72\%$

   $\sigma_P = 1.2727 \times 14.85\% = 18.90\%$
The investor’s utility level would be:

\[ U(P) = r_f + 10.72 - (0.005 \times 3 \times 18.90^2) = r_f + 5.36 \]

If borrowing is not allowed, investing 100% in Unicorn would lead to:

\[ E(r_P) - r_f = 8.42\% \]
\[ \sigma_P = 14.85\% \]

\[ U(P) = r_f + 8.42 - (0.005 \times 3 \times 14.85^2) = r_f + 5.11 \]

Note that, if Bull must be chosen, then:

\[ y_B = \frac{11.16}{0.01 \times 3 \times 21.24^2} = 0.8246 \]

\[ E(r_P) - r_f = 0.8246 \times 11.16\% = 9.20\% \]
\[ \sigma_P = 0.8246 \times 21.24\% = 17.51\% \]

\[ U(P) = r_f + 9.20 - (0.005 \times 3 \times 17.51^2) = r_f + 4.60 \]

Thus, even with a borrowing restriction, Unicorn (with the lower mean) is still superior to Bull.

2. Write the Black-Scholes formula from Chapter 21 as:

\[ C = S_0 N(d_1) - PV(X) N(d_2) \]

In this application, where we express the value of timing per dollar of assets, we use \( S_0 = 1.0 \) for the value of the stock. The present value of the exercise price is also equal to 1. Note that:

\[ d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}} \]
\[ d_2 = d_1 - \sigma \sqrt{T} \]

When \( S_0 = PV(X) \) and \( T = 1 \), the formula for \( d_1 \) reduces to: \( d_1 = \sigma / 2 \)

The formula for \( d_2 \) becomes: \( d_2 = -\sigma / 2 \)

Therefore: \( C = N(\sigma / 2) - N(-\sigma / 2) \)

Finally, recall that: \( N(-x) = 1 - N(x) \)

Therefore, we can write the value of the call as:

\[ C = N(\sigma / 2) - [1 - N(\sigma / 2)] = 2N(\sigma / 2) - 1 \]

Since \( \sigma = 0.055 \), the value of the option is:

\[ C = 2N(0.0275) - 1 \]

Interpolating from the standard normal table in Chapter 21:
\[ C = 2 \left[ 0.5080 + \left( \frac{0.0075}{0.0200} \times (0.5160 - 0.5080) \right) \right] - 1 = 1.0220 = 0.0220 \]

Hence the added value of a perfect timing strategy is 2.2% per month.

3. a. Using the relative frequencies to estimate the conditional probabilities \( P_1 \) and \( P_2 \) for timers A and B, we find:

<table>
<thead>
<tr>
<th>Timer</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timer A</td>
<td>78/135 = 0.58</td>
</tr>
<tr>
<td>Timer B</td>
<td>57/92 = 0.62</td>
</tr>
</tbody>
</table>

\[ P^* = P_1 + P_2 - 0.20 = 0.62 \]

The data suggest that timer A is the better forecaster.

b. Using the following equation to value the imperfect timing services of Timer A and Timer B:

\[ C(P^*) = C(P_1 + P_2 - 1) \]

\[ C_A(P^*) = 2.2\% \times 0.20 = 0.44\% \text{ per month} \]

\[ C_B(P^*) = 2.2\% \times 0.18 = 0.40\% \text{ per month} \]

Timer A’s added value is greater by 4 basis points per month.

4. a. 

\[ \alpha_i = r_i - \left[ r_f + \beta_i (r_M - r_f) \right] \]

\[ E(r_i) - r_f \]

\[ \alpha_A = 20\% - \left[ 8\% + 1.3(16\% - 8\%) \right] = 1.6\% \]

\[ 20\% - 8\% = 12\% \]

\[ \alpha_B = 18\% - \left[ 8\% + 1.8(16\% - 8\%) \right] = -4.4\% \]

\[ 18\% - 8\% = 10\% \]

\[ \alpha_C = 17\% - \left[ 8\% + 0.7(16\% - 8\%) \right] = 3.4\% \]

\[ 17\% - 8\% = 9\% \]

\[ \alpha_D = 12\% - \left[ 8\% + 1.0(16\% - 8\%) \right] = -4.0\% \]

\[ 12\% - 8\% = 4\% \]

Stocks A and C have positive alphas, whereas stocks B and D have negative alphas.

The residual variances are:

\[ \sigma^2(e_A) = 58^2 = 3,364 \]

\[ \sigma^2(e_B) = 71^2 = 5,041 \]

\[ \sigma^2(e_C) = 60^2 = 3,600 \]

\[ \sigma^2(e_D) = 55^2 = 3,025 \]

27-3
b. To construct the optimal risky portfolio, we first determine the optimal active portfolio. Using the Treynor-Black technique, we construct the active portfolio:

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\alpha}{\sigma^2(e)}$</th>
<th>$\frac{\alpha}{\Sigma\alpha / \sigma^2(e)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.000476</td>
<td>-0.6142</td>
</tr>
<tr>
<td>B</td>
<td>-0.000873</td>
<td>1.1265</td>
</tr>
<tr>
<td>C</td>
<td>0.000944</td>
<td>-1.2181</td>
</tr>
<tr>
<td>D</td>
<td>-0.001322</td>
<td>1.7058</td>
</tr>
<tr>
<td>Total</td>
<td>-0.000775</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Do not be concerned that the positive alpha stocks have negative weights and vice versa. We will see that the entire position in the active portfolio will be negative, returning everything to good order.

With these weights, the forecast for the active portfolio is:

$$\alpha = [-0.6142 \times 1.6] + [1.1265 \times (-4.4)] - [1.2181 \times 3.4] + [1.7058 \times (-4.0)] = -16.90\%$$

$$\beta = [-0.6142 \times 1.3] + [1.1265 \times 1.8] - [1.2181 \times 0.70] + [1.7058 \times 1] = 2.08$$

The high beta (higher than any individual beta) results from the short positions in the relatively low beta stocks and the long positions in the relatively high beta stocks.

$$\sigma^2(e) = [(-0.6142)^2 \times 3364] + [1.1265^2 \times 5041] + [(-1.2181)^2 \times 3600] + [1.7058^2 \times 3025] = 21,809.6$$

$$\sigma(e) = 147.68\%$$

Here, again, the levered position in stock B [with high $\sigma^2(e)$] overcomes the diversification effect, and results in a high residual standard deviation. The optimal risky portfolio has a proportion $w^*$ in the active portfolio, computed as follows:

$$w_0 = \frac{\alpha / \sigma^2(e)}{[E(r_M) - r_f] / \sigma_M^2} = \frac{-16.90}{21,809.6} = -0.05124$$

The negative position is justified for the reason stated earlier. The adjustment for beta is:

$$w^* = \frac{w_0}{1 + (1 - \beta)w_0} = \frac{-0.05124}{1 + (1 - 2.08)(-0.05124)} = -0.0486$$

Since $w^*$ is negative, the result is a positive position in stocks with positive alphas and a negative position in stocks with negative alphas. The position in
the index portfolio is:
\[ 1 - (-0.0486) = 1.0486 \]
c. To calculate Sharpe’s measure for the optimal risky portfolio, we compute the information ratio for the active portfolio and Sharpe’s measure for the market portfolio. The information ratio for the active portfolio is computed as follows:
\[ A = \frac{\alpha}{\sigma(e)} = \frac{-16.90}{147.68} = -0.1144 \]
\[ A^2 = 0.0131 \]
Hence, the square of Sharpe’s measure (S) of the optimized risky portfolio is:
\[ S^2 = S_M^2 + A^2 = \left( \frac{8}{23} \right)^2 + 0.0131 = 0.1341 \]
\[ S = 0.3662 \]
Compare this to the market’s Sharpe measure:
\[ S_M = \frac{8}{23} = 0.3478 \]
The difference is: 0.0184
Note that the only-moderate improvement in performance results from the fact that only a small position is taken in the active portfolio A because of its large residual variance.
We calculate the “Modigliani-squared” \( (M^2) \) measure, as follows:
\[ E(r_P) = r_f + S_P\sigma_M = 8\% + (0.3662 \times 23\%) = 16.423\% \]
\[ M^2 = E(r_P) - E(r_M) = 16.423\% - 16\% = 0.423\% \]
d. To calculate the exact makeup of the complete portfolio, we first compute the mean excess return of the optimal risky portfolio and its variance. The risky portfolio beta is given by:
\[ \beta_p = w_M + (w_A \times \beta_A) = 1.0486 + \left( -0.0486 \times 2.08 \right) = 0.95 \]
\[ E(R_P) = \alpha_p + \beta_p E(R_M) = \left[ (-0.0486) \times (-16.90\%) \right] + (0.95 \times 8\%) = 8.42\% \]
\[ \sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p) = (0.95 \times 23)^2 + \left( -0.0486^2 \times 21,809.6 \right) = 528.94 \]
\[ \sigma_p = 23.00\% \]
Since \( A = 2.8 \), the optimal position in this portfolio is:
\[ y = \frac{8.42}{0.01 \times 2.8 \times 528.94} = 0.5685 \]
In contrast, with a passive strategy:
This is a difference of: 0.0284

The final positions of the complete portfolio are:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bills</td>
<td>$1 - 0.5685 = 43.15%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>$0.5685 \times 1.0486 = 59.61%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>$0.5685 \times (-0.0486) \times (-0.6142) = 1.70%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>$0.5685 \times (-0.0486) \times 1.1265 = -3.11%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>$0.5685 \times (-0.0486) \times (-1.2181) = 3.37%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$0.5685 \times (-0.0486) \times 1.7058 = -4.71%$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$100.00%$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[sum is subject to rounding error]

Note that M may include positive proportions of stocks A through D.

5. a. If a manager is not allowed to sell short he will not include stocks with negative alphas in his portfolio, so he will consider only A and C:

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\sigma^2(e)$</th>
<th>$\frac{\alpha}{\sigma^2(e)}$</th>
<th>$\frac{\alpha}{\sigma^2(e)} \Sigma \alpha / \sigma^2(e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.6</td>
<td>3,364</td>
<td>0.000476</td>
<td>0.3352</td>
</tr>
<tr>
<td>C</td>
<td>3.4</td>
<td>3,600</td>
<td>0.000944</td>
<td>0.6648</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.001420</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

The forecast for the active portfolio is:

\[
\alpha = (0.3352 \times 1.6) + (0.6648 \times 3.4) = 2.80\%
\]

\[
\beta = (0.3352 \times 1.3) + (0.6648 \times 0.7) = 0.90
\]

\[
\sigma^2(e) = (0.3352^2 \times 3,364) + (0.6648^2 \times 3,600) = 1,969.03
\]

\[
\sigma(e) = 44.37\%
\]

The weight in the active portfolio is:

\[
w_0 = \frac{\alpha / \sigma^2(e)}{E(R_M) / \sigma^2_M} = \frac{2.80 / 1,969.03}{8 / 23^2} = 0.0940
\]

Adjusting for beta:

\[
w^* = \frac{w_0}{1 + (1 - \beta)w_0} = \frac{0.094}{1 + [(1 - 0.90) \times 0.094]} = 0.0931
\]

The information ratio of the active portfolio is:
\[ A = \frac{\alpha}{\sigma(e)} = \frac{2.80}{44.37} = 0.0631 \]

Hence, the square of Sharpe’s measure is:
\[ S^2 = \left(\frac{8}{23}\right)^2 + 0.0631^2 = 0.1250 \]

Therefore: \( S = 0.3535 \)

The market’s Sharpe measure is: \( S_M = 0.3478 \)

When short sales are allowed (Problem 4), the manager’s Sharpe measure is higher (0.3662). The reduction in the Sharpe measure is the cost of the short sale restriction.

We calculate the “Modigliani-squared” or \( M^2 \) measure as follows:
\[ E(r_{P*}) = r_f + S_P\sigma_M = 8\% + (0.3535 \times 23\%) = 16.1305\% \]
\[ M^2 = E(r_{P*}) - E(r_M) = 16.1305\% - 16\% = 0.1305\% \]

When short sales are allowed: \( M^2 = 0.423\% \)

The characteristics of the optimal risky portfolio are:
\[ \beta_P = w_M + w_A \times \beta_A = (1 - 0.0931) + (0.0931 \times 0.9) = 0.99 \]
\[ E(R_P) = \alpha_p + \beta_P E(R_M) = (0.0931 \times 2.8\%) + (0.99 \times 8\%) = 8.18\% \]
\[ \sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma^2(e_p) = (0.99 \times 23)^2 + (0.0931^2 \times 1.969.03) = 535.54 \]
\[ \sigma_P = 23.14\% \]

With \( A = 2.8 \), the optimal position in this portfolio is:
\[ y = \frac{8.18}{0.01 \times 2.8 \times 535.54} = 0.5455 \]

The final positions in each asset are:

<table>
<thead>
<tr>
<th>Asset</th>
<th>( w_P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bills</td>
<td>( 1 - 0.5455 = 45.45% )</td>
</tr>
<tr>
<td>M</td>
<td>( 0.5455 \times (1 - 0.0931) = 49.47% )</td>
</tr>
<tr>
<td>A</td>
<td>( 0.5455 \times 0.0931 \times 0.3352 = 1.70% )</td>
</tr>
<tr>
<td>C</td>
<td>( 0.5455 \times 0.0931 \times 0.6648 = 3.38% )</td>
</tr>
<tr>
<td></td>
<td>100.00%</td>
</tr>
</tbody>
</table>

b. The mean and variance of the optimized complete portfolios in the unconstrained and short-sales constrained cases, and for the passive strategy are:

<table>
<thead>
<tr>
<th></th>
<th>( E(R_C) )</th>
<th>( \sigma_C^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>0.5685 \times 8.42 = 4.79</td>
<td>0.5685^2 \times 528.94 = 170.95</td>
</tr>
</tbody>
</table>

27-7
Constrained $0.5455 \times 8.18 = 4.46 \quad 0.5455^2 \times 535.54 = 159.36$
Passive $0.5401 \times 8.00 = 4.32 \quad 0.5401^2 \times 529.00 = 154.31$

The utility levels below are computed using the formula: $E(r_c) - 0.005A \sigma_c^2$

Unconstrained $8 + 4.79 - (0.005 \times 2.8 \times 170.95) = 10.40$
Constrained $8 + 4.46 - (0.005 \times 2.8 \times 159.36) = 10.23$
Passive $8 + 4.32 - (0.005 \times 2.8 \times 154.31) = 10.16$

6. a. The optimal passive portfolio is obtained from equation (8.7) in Chapter 8:

Optimal Risky Portfolios:

$$w_M = \frac{E(R_M)\sigma_H^2 - E(R_H)\text{Cov}(r_H, r_M)}{E(R_M)\sigma_H^2 + E(R_H)\sigma_M^2 - [E(R_H) + E(R_M)\text{Cov}(r_H, r_M)]}$$

$E(R_M) = 8\%$, $E(R_H) = 2\%$ and

$\text{Cov}(r_H, r_M) = \rho \sigma_M \sigma_H = 0.6 \times 23 \times 18 = 248.4$

$$w_M = \frac{(8 \times 18^2) - (2 \times 248.4)}{(8 \times 18^2) + (2 \times 23^2) - [(8 + 2) \times 248.4]} = 1.797$$

$$w_H = -0.797$$

If short sales are not allowed, portfolio H would have to be omitted from the passive portfolio because $w_H$ is negative.

b. With short sales allowed:

$E(R_{\text{passive}}) = (1.797 \times 8\%) + [(-0.797) \times 2\%] = 12.78\%$

$\sigma_{\text{passive}}^2 = (1.797 \times 23)^2 + [(-0.797) \times 18]^2 + [2 \times 1.797 \times (-0.797) \times 248.4] = 1,202.54$

$\sigma_{\text{passive}} = 34.68\%$

Sharpe’s measure in this case is: $S_{\text{passive}} = 12.78/34.68 = 0.3685$

The market’s Sharpe measure is: $S_M = 8/23 = 0.3478$

c. The improvement in utility for the expanded model of H and M versus a portfolio of M alone, for $A = 2.8$, is:

$$y = \frac{12.78}{0.01 \times 2.8 \times 1,202.54} = 0.3796$$

27-8
Therefore:

\[ U_{\text{passive}} = 8 + (12.78 \times 0.3796) - (0.005 \times 2.8 \times 0.3796^2 \times 1,202.54) = 10.43 \]

This result is greater than \( U_{\text{passive}} = 10.16 \) from Problem 5.

7. The first step is to find the beta of the stocks relative to the optimized passive portfolio. For any stock \( i \), the covariance with a portfolio is the sum of the covariances with the portfolio components, accounting for the weights of the components. Thus:

\[
\beta_i = \frac{\text{Cov}(r_i, r_{\text{passive}})}{\sigma_{\text{passive}}^2} = \beta_{i,M} w_M \sigma_M^2 + \beta_{i,H} w_H \sigma_H^2
\]

Therefore:

\[
\beta_A = \frac{(1.2 \times 1.797 \times 23^2) + (1.8 \times (-0.797) \times 18^2)}{1,202.54} = 0.5621
\]

\[
\beta_B = \frac{(1.4 \times 1.797 \times 23^2) + (1.1 \times (-0.797) \times 18^2)}{1,202.54} = 0.8705
\]

\[
\beta_C = \frac{(0.5 \times 1.797 \times 23^2) + (1.5 \times (-0.797) \times 18^2)}{1,202.54} = 0.0731
\]

\[
\beta_D = \frac{(1.0 \times 1.797 \times 23^2) + (0.2 \times (-0.797) \times 18^2)}{1,202.54} = 0.7476
\]

The alphas relative to the optimized portfolio are:

\[
\alpha_i = E(r_i) - r_f - [\beta_i, \text{passive} \times E(R_{\text{passive}})]
\]

\[
\alpha_A = 20 - 8 - (0.5621 \times 12.78) = 4.82\%
\]

\[
\alpha_B = 18 - 8 - (0.8705 \times 12.78) = -1.12\%
\]

\[
\alpha_C = 17 - 8 - (0.0731 \times 12.78) = 8.07\%
\]

\[
\alpha_D = 12 - 8 - (0.7476 \times 12.78) = -5.55\%
\]

The residual variances are now obtained from:

\[
\sigma^2(e_i, \text{passive}) = \sigma_i^2 - (\beta_i, \text{passive} \times \sigma_{\text{passive}}^2)
\]

where: \( \sigma_i^2 = \beta_M^2 \sigma_M^2 + \sigma^2(e_i) \) from Problem 4.

\[
\sigma^2(e_A) = (1.3 \times 23)^2 + 58^2 - (0.5621 \times 34.68)^2 = 3,878.01
\]

\[
\sigma^2(e_B) = (1.8 \times 23)^2 + 71^2 - (0.8705 \times 34.68)^2 = 5,843.59
\]
\[ \sigma^2(e_C) = (0.7 \times 23)^2 + 60^2 - (0.0731 \times 34.68)^2 = 3,852.78 \]

\[ \sigma^2(e_D) = (1.0 \times 23)^2 + 55^2 - (0.7476 \times 34.68)^2 = 2,881.80 \]

From this point, the procedure is identical to that of Problem 6:

<table>
<thead>
<tr>
<th></th>
<th>(\frac{\alpha}{\sigma^2(e)})</th>
<th>(\frac{\alpha}{\sigma^2(eA)})</th>
<th>(\Sigma\alpha / \sigma^2(eA))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.001243</td>
<td>1.0189</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>-0.000192</td>
<td>-0.1574</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.002095</td>
<td>1.7172</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-0.001926</td>
<td>-1.5787</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.001220</td>
<td>1.0000</td>
<td></td>
</tr>
</tbody>
</table>

The active portfolio parameters are:

\[ \alpha = (1.0189 \times 4.82) + [(-0.1574) \times (-1.12)] + (1.7172 \times 8.07) + [(-1.5787) \times (-5.55)] = 27.71\% \]

\[ \beta = (1.0189 \times 0.5621) + [(-0.1574) \times 0.8705] + (1.7172 \times 0.0731) + [(-1.5787) \times 0.7476) = -0.6190 \]

\[ \sigma^2(e) = (1.0189^2 \times 3,878.01) + [(-0.1574)^2 \times 5,843.59] + (1.7172^2 \times 3,852.78) + [(-1.5787)^2 \times 2881.80] = 22,714.03 \]

The proportions in the overall risky portfolio can now be determined:

\[ w_0 = \frac{\alpha / \sigma^2(e)}{E(R_{\text{passive}}) / \sigma^2_{\text{passive}}} = \frac{27.71/22,714.03}{12.78/1,202.54} = 0.1148 \]

\[ w^* = \frac{w_0}{1 + (1 - \beta)w_0} = \frac{0.1148}{1 + [(1 + 0.6190) \times 0.1148]} = 0.0968 \]

a. Sharpe’s measure for the optimal risky portfolio is

\[ S^2 = S^2_{\text{passive}} + \left[ \frac{\alpha}{\sigma^2(e)} \right]^2 = 0.3685^2 + \frac{27.71^2}{22,714.03} = 0.1696 \]

\[ S = 0.4118 \text{ compared to } S_{\text{passive}} = 0.3685 \]

Therefore, the difference in the Sharpe measure is: 0.0433

b. The beta of the optimal risky portfolio is:

\[ \beta_p = w^* \beta_A + (1 - w^*) = [0.0968 \times (-0.6190)] + 0.9032 = 0.8433 \]

The mean excess return of this portfolio is:

27-10
\[
E(R) = (0.0968 \times 27.71\%) + (0.8433 \times 12.78\%) = 13.46\%
\]
The variance and standard deviation are:
\[
\sigma^2 = 0.8433^2 \times 1.202.54 + 0.0968^2 \times 22,714.03 = 1,068.03
\]
\[
\sigma = 32.68\%
\]
Therefore, the position in the optimal risky portfolio would be:
\[
y = \frac{13.46}{0.01 \times 2.8 \times 1,068.03} = 0.4501
\]
The utility value for this portfolio is:
\[
U = 8 + (0.4501 \times 13.46) - (0.005 \times 2.8 \times 0.4501^2 \times 1,068.03) = 11.03
\]
This value is superior to all previous alternatives.

8. If short sales are not allowed, then the passive portfolio reverts to \( M \), and the solution mimics the solution to Problem 5.

9. All alphas are reduced to 0.3 times their values in the original case. Therefore, the relative weights of each security in the active portfolio are unchanged, but the alpha of the active portfolio is only 0.3 times its previous value: 0.3 \times -16.90\% = -5.07\%
The investor will take a smaller position in the active portfolio. The optimal risky portfolio has a proportion \( w^* \) in the active portfolio as follows:
\[
w_0 = \frac{\alpha / \sigma^2(e)}{E(r_M - r_f) / \sigma_M^2} = -\frac{5.07}{21,809.6} \times \frac{8}{23^2} = -0.01537
\]
The negative position is justified for the reason given earlier.
The adjustment for beta is:
\[
w^* = \frac{w_0}{1 + (1 - \beta)w_0} = \frac{-0.01537}{1 + [(1 - 2.08) \times (-0.01537)]} = -0.0151
\]
Since \( w^* \) is negative, the result is a positive position in stocks with positive alphas and a negative position in stocks with negative alphas. The position in the index portfolio is:
\[
1 - (-0.0151) = 1.0151
\]
To calculate Sharpe’s measure for the optimal risky portfolio we compute the information ratio for the active portfolio and Sharpe’s measure for the market portfolio. The information ratio of the active portfolio is 0.3 times its previous value:
\[
A = \alpha / \sigma(e) = -5.07 / 147.68 = -0.0343 \text{ and } A^2 = 0.00118
\]
Hence, the square of Sharpe’s measure of the optimized risky portfolio is:

\[ S^2 = S_M^2 + A^2 = (8/23)^2 + 0.00118 = 0.1222 \]

\[ S = 0.3495 \]

Compare this to the market’s Sharpe measure: \( S_M = 8/23 = 0.3478 \)

The difference is: 0.0017

Note that the reduction of the forecast alphas by a factor of 0.3 reduced the squared information ratio and the improvement in the squared Sharpe ratio by a factor of:

\[ 0.3^2 = 0.09 \]