CHAPTER 6: RISK AND RISK AVERSION

1. a. The expected cash flow is: $(0.5 \times \$70{,}000) + (0.5 \times 200{,}000) = \$135{,}000$

   With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

   $\$135{,}000/1.14 = \$118{,}421$

   b. If the portfolio is purchased for \$118,421, and provides an expected cash inflow of \$135,000, then the expected rate of return [E(r)] is derived as follows:

   $\$118{,}421 \times [1 + E(r)] = \$135{,}000$

   Therefore, E(r) = 14%. The portfolio price is set to equate the expected rate or return with the required rate of return.

   c. If the risk premium over T-bills is now 12%, then the required return is:

   $6\% + 12\% = 18\%$

   The present value of the portfolio is now:

   $\$135{,}000/1.18 = \$114{,}407$

   d. For a given expected cash flow, portfolios that command greater risk premia must sell at lower prices. The extra discount from expected value is a penalty for risk.

2. When we specify utility by $U = E(r) - 0.005A\sigma^2$, the utility level for T-bills is 7%. The utility level for the risky portfolio is: $U = 12 - 0.005A \times 18^2 = 12 - 1.62A$
   In order for the risky portfolio to be preferred to bills, the following inequality must hold:

   $12 - 1.62A > 7 \Rightarrow A < 5/1.62 = 3.09$

   A must be less than 3.09 for the risky portfolio to be preferred to bills.

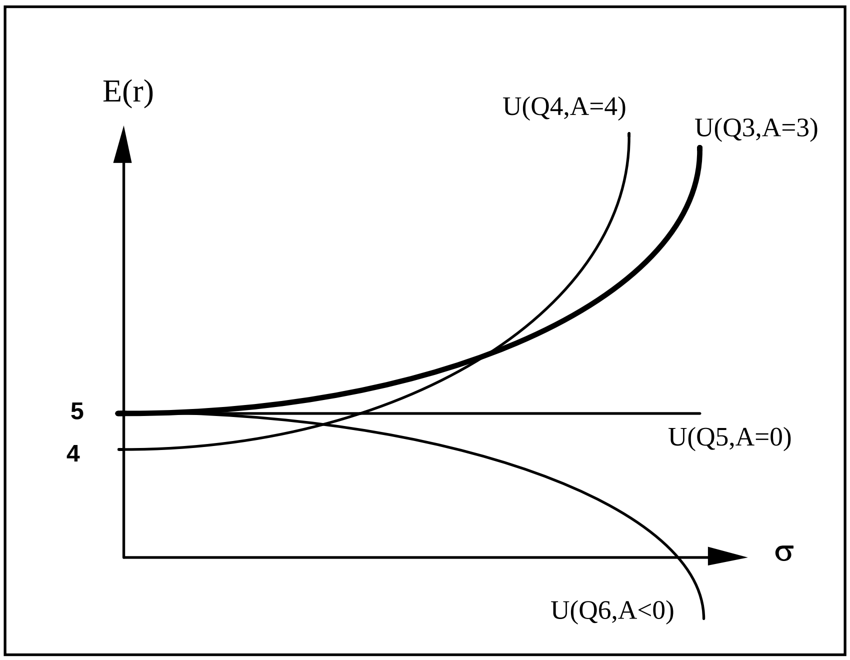
3. Points on the curve are derived by solving for E(r) in the following equation:

$$U = 5 = E(r) - 0.005A\sigma^2 = E(r) - 0.015\sigma^2$$

The values of E(r), given the values of σ^2, are therefore:

σ	σ^2	E(r)
0%	0	5.000%
5%	25	5.375%
10%	100	6.500%
15%	225	8.375%
20%	400	11.000%
25%	625	14.375%

The bold line in the following graph (labeled Q3, for Question 3) depicts the indifference curve.



4. Repeating the analysis in Problem 3, utility is now:

$$U = E(r) - 0.005A\sigma^2 = E(r) - 0.020\sigma^2 = 4$$

The equal-utility combinations of expected return and standard deviation are presented in the table below. The indifference curve is the upward sloping line in the graph above, labeled Q4 (for Question 4).

σ	σ^2	E(r)
0%	0	4.000%
5%	25	4.500%
10%	100	6.000%
15%	225	8.500%
20%	400	12.000%
25%	625	16.500%

The indifference curve in Problem 4 differs from that in Problem 3 in both slope and intercept. When A increases from 3 to 4, the increased risk aversion results in a greater slope for the indifference curve since more expected return is needed in order to compensate for additional σ. The lower level of utility assumed for Problem 4 (4% rather than 5%) shifts the vertical intercept down by 1%.

5. The coefficient of risk aversion for a risk neutral investor is zero. Therefore, the corresponding utility is equal to the portfolio's expected return. The corresponding indifference curve in the expected return-standard deviation plane is a horizontal line, labeled Q5 in the graph above (see Problem 3).

6. A risk lover, rather than penalizing portfolio utility to account for risk, derives greater utility as variance increases. This amounts to a negative coefficient of risk aversion. The corresponding indifference curve is downward sloping in the graph above (see Problem 3), and is labeled Q6.

7. c [Utility for each portfolio = $E(r) - 0.005 \times 4 \times \sigma^2$
We choose the portfolio with the highest utility value.]

8. d [When investors are risk neutral, then A = 0; the portfolio with the highest utility is the one with the highest expected return.]

9. b

10. The portfolio expected return and variance are computed as follows:

(1) W_{Bills}	(2) r_{Bills}	(3) W_{Index}	(4) r_{Index}	$r_{Portfolio}$ (1)×(2)+(3)×(4)	$\sigma_{Portfolio}$ (3) × 20%	$\sigma^2_{Portfolio}$
0.0	5%	1.0	13.5%	13.5%	20%	400
0.2	5%	0.8	13.5%	11.8%	16%	256
0.4	5%	0.6	13.5%	10.1%	12%	144
0.6	5%	0.4	13.5%	8.4%	8%	64
0.8	5%	0.2	13.5%	6.7%	4%	16
1.0	5%	0.0	13.5%	5.0%	0%	0

11. Computing utility from $U = E(r) - 0.005 \times A\sigma^2 = E(r) - 0.015\sigma^2$, we arrive at the values in the column labeled U(A = 3) in the following table:

W_{Bills}	W_{Index}	$r_{Portfolio}$	$\sigma_{Portfolio}$	$\sigma^2_{Portfolio}$	U(A = 3)	U(A = 5)
0.0	1.0	13.5%	20%	400	7.50	3.50
0.2	0.8	11.8%	16%	256	7.96	5.40
0.4	0.6	10.1%	12%	144	7.94	6.50
0.6	0.4	8.4%	8%	64	7.44	6.80
0.8	0.2	6.7%	4%	16	6.46	6.30
1.0	0.0	5.0%	0%	0	5.00	5.00

The column labeled U(A = 3) implies that investors with A = 3 prefer a portfolio that is invested 80% in the market index and 20% in T-bills to any of the other portfolios in the table.

12. The column labeled U(A = 5) in the table above is computed from:

$$U = E(r) - 0.005\ A\sigma^2 = E(r) - 0.025\sigma^2$$

The more risk averse investors prefer the portfolio that is invested 40% in the market index, rather than the 80% market weight preferred by investors with A = 3.

13. Sugarcane is now less useful as a hedge. The probability distribution is as follows:

	Normal Year for Sugar		Abnormal Year
	Bullish Stock Market	Bearish Stock Market	
Probability	0.5	0.3	0.2
Stock			
Best Candy	25.0%	10.0%	–25.0%
Sugarcane	10.0%	–5.0%	20.0%
Humanex's Portfolio	17.5%	2.5%	–2.5%

Using the distribution of portfolio rate of return, the expected return and standard deviation are calculated as follows:

$$E(r_p) = (0.5 \times 17.5) + (0.3 \times 2.5) + [0.2 \times (-2.5)] = 9.0\%$$

$$\sigma_p = [0.5 \times (17.5 - 9)^2 + 0.3 \times (2.5 - 9)^2 + [0.2 \times (-2.5 - 9)^2]^{1/2} = 8.67\%$$

While the expected return has improved somewhat, the standard deviation is now significantly greater, and only marginally better than investing half of the portfolio in T-bills.

14. The expected return for Best Candy is 10.5% and the standard deviation is 18.9%. The mean and standard deviation for Sugarcane are now:

$E(r) = (0.5 \times 10) + [0.3 \times (–5)] + (0.2 \times 20) = 7.5\%$

$\sigma = [\ 0.5 \times (10 – 7.5)^{\ 2} – 0.3 \times (–5 – 7.5)^{\ 2} + 0.2 \times (20 – 7.5)^{\ 2}]^{\ 1/2} = 9.01\%$

The covariance between Best Candy and Sugarcane is:

$Cov(r_{Best}, r_{Cane}) =$

$[0.5(25 – 10.5)(10 – 7.5)] + [0.3(10 – 10.5)(–5 – 7.5)] + [0.2(–25 – 10.5)(20 –7.5)] = –68.75$

15. Using the results from Problem 14, the portfolio expected rate of return is computed as follows:

$E(r_p) = (0.5 \times 10.5) + (0.5 \times 7.5) = 9\%$

We can use Rule 5 to compute the portfolio standard deviation as follows:

$$\sigma_P = [w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_B w_C Cov(r_B, r_C)]^{1/2}$$

$$= [(0.5^2 \times 18.9^2) + (0.5^2 \times 9.01^2) + (2 \times 0.5 \times 0.5 \times (–68.75))]^{1/2} = 8.67\%$$

CHAPTER 6: APPENDIX A

1. The price of Klink stock is $12 per share. The rate of return in each scenario is shown in the following table:

Probability	Rate of Return (%)
0.10	–100.000
0.20	–81.250
0.40	20.000
0.25	71.667
0.05	157.083

a.

Probability	Rate of Return (%)	Probability × Rate of Return	Deviation from Mean (%)
0.10	–100.000	–10.00000	–107.5209
0.20	–81.250	–16.25000	–88.7709
0.40	20.000	8.00000	12.4791
0.25	71.667	17.91675	64.1461
0.05	157.083	7.85415	149.5621
	Mean =	7.52090%	
	Median =	20.00%	
	Mode =	20.00%	

b.

Probability	Deviation from Mean (%)	Squared Deviation	Probability × Squared Deviation	Probability × Absolute Deviation
0.10	–107.5209	11,560.7439	1,156.0744	10.7521
0.20	–88.7709	7,880.2727	1,576.0545	17.7542
0.40	12.4791	155.7279	62.2912	4.9916
0.25	64.1461	4,114.7221	1,028.6805	16.0365
0.05	149.5621	22,368.8218	1,118.4411	7.4781
		Variance =	4,941.5417	
		Standard Deviation =	70.2961%	
			Mean Absolute Deviation =	57.0125%

c. The first moment is the mean (7.5209%), the second moment around the mean is the variance (70.2961^2) and the third moment around the mean is:

$$M_3 = \Sigma_S \Pr(s)\,[r(s) - E(r)]^3 = -30{,}170.36$$

Therefore the probability distribution is negatively (left) skewed.

CHAPTER 6: APPENDIX B

1. By year end, the \$50,000 investment will grow to: \$50,000 × 1.06 = \$53,000
Without insurance, the probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	\$253,000
Fire	0.001	\$ 53,000

For this distribution, expected utility is computed as follows:

$$E[U(W)] = [0.999 \times \ln(253{,}000)] + [0.001 \times \ln(53{,}000)] = 12.439582$$

The certainty equivalent is:

$$W_{CE} = e^{12.439582} = \$252{,}604.85$$

With fire insurance, at a cost of \$P, the investment in the risk-free asset is:

$$\$(50{,}000 - P)$$

Year-end wealth will be certain (since you are fully insured) and equal to:

$$[\$(50{,}000 - P) \times 1.06] + \$200{,}000$$

Solve for P in the following equation:

$$[\$(50{,}000 - P) \times 1.06] + \$200{,}000 = \$252{,}604.85 \Rightarrow P = \$372.78$$

This is the most you are willing to pay for insurance. Note that the expected loss is “only” \$200, so you are willing to pay a substantial risk premium over the expected value of losses. The primary reason is that the value of the house is a large proportion of your wealth.

2. a. With insurance coverage for one-half the value of the house, the premium is \$100, and the investment in the safe asset is \$49,900. By year end, the investment of \$49,900 will grow to: \$49,900 × 1.06 = \$52,894
If there is a fire, your insurance proceeds will be \$100,000, and the probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	\$252,894
Fire	0.001	\$152,894

For this distribution, expected utility is computed as follows:

$$E[U(W)] = [0.999 \times \ln(252{,}894)] + [0.001 \times \ln(152{,}894)] = 12.4402225$$

The certainty equivalent is:

$$W_{CE} = e^{12.4402225} = \$252{,}766.77$$

b. With insurance coverage for the full value of the house, costing \$200, end-of-year wealth is certain, and equal to:

$$[(\$50{,}000 - \$200) \times 1.06] + \$200{,}000 = \$252{,}788$$

Since wealth is certain, this is also the certainty equivalent wealth of the fully insured position.

c. With insurance coverage for 1½ times the value of the house, the premium is \$300, and the insurance pays off \$300,000 in the event of a fire. The investment in the safe asset is \$49,700. By year end, the investment of \$49,700 will grow to: \$49,700 × 1.06 = \$52,682
The probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	\$252,682
Fire	0.001	\$352,682

For this distribution, expected utility is computed as follows:

$$E[U(W)] = [0.999 \times \ln(252{,}682)] + [0.001 \times \ln(352{,}682)] = 12.4402205$$

The certainty equivalent is:

$$W_{CE} = e^{12.440222} = \$252{,}766.27$$

Therefore, full insurance dominates both over- and under-insurance. Over-insuring creates a gamble (you actually gain when the house burns down). Risk is minimized when you insure exactly the value of the house.