CHAPTER 7: CAPITAL ALLOCATION BETWEEN THE RISKY ASSET AND THE RISK-FREE ASSET

1. Expected return = $(0.7 \times 18\%) + (0.3 \times 8\%) = 15\%$ Standard deviation = $0.7 \times 28\% = 19.6\%$

Investment proportions:

2.

	30.0% in T-bills
0.7 × 25% =	17.5% in Stock A
0.7 × 32% =	22.4% in Stock B
0.7 × 43% =	30.1% in Stock C

3. Your reward-to-variability ratio: $S = \frac{18-8}{28} = 0.3571$ Client's reward-to-variability ratio: $S = \frac{15-8}{19.6} = 0.3571$



5. a. $E(r_C) = r_f + y[E(r_P) - r_f] = 8 + y(18 - 8)$

If the expected return for the portfolio is 16%, then:

$$16 = 8 + 10 \text{ y} \Rightarrow \text{y} = \frac{16 - 8}{10} = 0.8$$

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

Client's investment proportions:	20.0% in T-bills
$0.8 \times 25\% =$	20.0% in Stock A
$0.8 \times 32\% =$	25.6% in Stock B
$0.8 \times 43\% =$	34.4% in Stock C

c. $\sigma_{C} = 0.8 \times \sigma_{P} = 0.8 \times 28\% = 22.4\%$

6. a.
$$\sigma_{\rm C} = y \times 28\%$$

If your client prefers a standard deviation of at most 18%, then: y = 18/28 = 0.6429 = 64.29% invested in the risky portfolio

b.
$$E(r_C) = 8 + 10y = 8 + (0.6429 \times 10) = 8 + 6.429 = 14.429\%$$

7. a.
$$y^* = \frac{E(r_p) - r_f}{0.01A\sigma_p^2} = \frac{18 - 8}{0.01 \times 3.5 \times 28^2} = \frac{10}{27.44} = 0.3644$$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 8 + 10y^* = 8 + (0.3644 \times 10) = 11.644\%$

 $\sigma_C = 0.3644 \times 28 = 10.203\%$

8. a. Slope of the CML =
$$\frac{13-8}{25} = 0.20$$

The diagram follows.

b. My fund allows an investor to achieve a higher mean for any given standard deviation than would a passive strategy, i.e., a higher expected return for any given level of risk.

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9. a. With 70% of his money invested in my fund's portfolio, the client's expected return is 15% per year and standard deviation is 19.6% per year. If he shifts that money to the passive portfolio (which has an expected return of 13% and standard deviation of 25%), his overall expected return becomes:

$$E(r_{c}) = r_{f} + 0.7[E(r_{M}) - r_{f}] = 8 + [0.7 \times (13 - 8)] = 11.5\%$$

The standard deviation of the complete portfolio using the passive portfolio would be:

 $\sigma_{C} = 0.7 \times \sigma_{M} = 0.7 \times 25\% = 17.5\%$

Therefore, the shift entails a decrease in mean from 14% to 11.5% and a decrease in standard deviation from 19.6% to 17.5%. Since both mean return *and* standard deviation decrease, it is not yet clear whether the move is beneficial. The disadvantage of the shift is that, if the client is willing to accept a mean return on his total portfolio of 11.5%, he can achieve it with a lower standard deviation using my fund rather than the passive portfolio.

To achieve a target mean of 11.5%, we first write the mean of the complete portfolio as a function of the proportion invested in my fund (y):

 $E(r_C) = 8 + y(18 - 8) = 8 + 10y$

Our target is: $E(r_c) = 11.5\%$. Therefore, the proportion that must be invested in my fund is determined as follows:

$$11.5 = 8 + 10y \Rightarrow y = \frac{11.5 - 8}{10} = 0.35$$

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The standard deviation of this portfolio would be:

$$\sigma_C = y \times 28\% = 0.35 \times 28\% = 9.8\%$$

Thus, by using my portfolio, the same 11.5% expected return can be achieved with a standard deviation of only 9.8% as opposed to the standard deviation of 17.5% using the passive portfolio.

b. The fee would reduce the reward-to-variability ratio, i.e., the slope of the CAL. The client will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee:

Slope of CAL with fee
$$=\frac{18-8-f}{28} = \frac{10-f}{28}$$

Slope of CML (which requires no fee) $=\frac{13-8}{25} = 0.20$

Setting these slopes equal we have:

$$\frac{10 - f}{28} = 0.20 \Rightarrow 10 - f = 28 \times 0.20 = 5.6 \Rightarrow f = 10 - 5.6 = 4.4\% \text{ per year}$$

10. a. The formula for the optimal proportion to invest in the passive portfolio is:

$$y^* = \frac{E(r_M) - r_f}{0.01A\sigma_M^2}$$

Substitute the following: $E(r_M) = 13\%$; $r_f = 8\%$; $\sigma_M = 25\%$; A = 3.5:

$$y^* = \frac{13 - 8}{0.01 \times 3.5 \times 25^2} = 0.2286$$

- b. The answer here is the same as the answer to Problem 9(b). The fee that you can charge a client is the same regardless of the asset allocation mix of the client's portfolio. You can charge a fee that will equalize the reward-to-variability *ratio* of your portfolio with that of your competition.
- 11. a. If the period 1926 2002 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: A = 4, $E(r_M) r_f = 8.22\%$, $\sigma_M = 20.81\%$ (we use the standard deviation of the risk premium from Table 7.4). Then y^{*} is given by:

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$$y^* = \frac{E(r_M) - r_f}{0.01A\sigma_M^2} = \frac{8.22}{0.01 \times 4 \times 20.81^2} = 0.4745$$

That is, 47.45% of the portfolio should be allocated to equity and 52.55% should be allocated to T-bills.

b. If the period 1983 - 2002 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: A = 4, $E(r_M) - r_f = 8.38\%$, $\sigma_M = 16.26\%$ and y^* is given by:

$$y^* = \frac{E(r_M) - r_f}{0.01A\sigma_M^2} = \frac{8.38}{0.01 \times 4 \times 16.26^2} = 0.7924$$

Therefore, 79.24% of the complete portfolio should be allocated to equity and 20.76% should be allocated to T-bills.

- c. In part (b), the market risk premium is expected to be higher than in part (a) and market risk is lower. Therefore, the reward-to-variability *ratio* is expected to be higher in part (b), which explains the greater proportion invested in equity.
- 12. Assuming no change in risk tolerance, that is, an unchanged risk aversion coefficient (A), then higher perceived volatility increases the denominator of the equation for the optimal investment in the risky portfolio (Equation 7.5). The proportion invested in the risky portfolio will therefore decrease.

13. a.
$$E(r_C) = 8\% = 5\% + y(11\% - 5\%) \Rightarrow y = \frac{8-5}{11-5} = 0.5$$

b.
$$\sigma_{\rm C} = y\sigma_{\rm P} = 0.50 \times 15\% = 7.5\%$$

- c. The first client is more risk averse, allowing a smaller standard deviation.
- 14. Data: $r_f = 5\%$, $E(r_M) = 13\%$, $\sigma_M = 25\%$, and $r_f^B = 9\%$

The CML and indifference curves are as follows:

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15. For *y* to be less than 1.0 (so that the investor is a lender), risk aversion (A) must be large enough such that:

$$y = \frac{E(r_{M}) - r_{f}}{0.01A\sigma_{M}^{2}} < 1 \implies A > \frac{13 - 5}{0.01 \times 25^{2}} = 1.28$$

For y to be greater than 1.0 (so that the investor is a borrower), risk aversion must be small enough such that:

$$y = \frac{E(r_M) - r_f}{0.01A\sigma_M^2} > 1 \implies A < \frac{13 - 9}{0.01 \times 25^2} = 0.64$$

For values of risk aversion within this range, the client will neither borrow nor lend, but instead will hold a complete portfolio comprised only of the optimal risky portfolio:

$$y = 1$$
 for $0.64 \le A \le 1.28$

16. a. The graph for Problem 14 has to be redrawn here, with: $E(r_P) = 11\% \text{ and } \sigma_P = 15\%$

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b. For a lending position: $A > \frac{11-5}{0.01 \times 15^2} = 2.67$ For a borrowing position: $A < \frac{11-5}{0.01 \times 15^2} = 0.89$

Therefore, y = 1 for $0.89 \le A \le 2.67$



17. The maximum feasible fee, denoted f, depends on the reward-to-variability ratio. For y < 1, the lending rate, 5%, is viewed as the relevant risk-free rate, and we solve for f as follows:

$$\frac{11-5-f}{15} = \frac{13-5}{25} \implies f = 6 - \frac{15 \times 8}{25} = 1.2\%$$

For y > 1, the borrowing rate, 9%, is the relevant risk-free rate. Then we notice that, even without a fee, the active fund is inferior to the passive fund because:

$$\frac{11-9}{15} = 0.13 < \frac{13-9}{25} = 0.16$$

More risk tolerant investors (who are more inclined to borrow) will not be clients of the fund even without a fee. (If you solved for the fee that would make investors who borrow indifferent between the active and passive portfolio, as we did above for lending investors, you would find that f is negative: that is, you would need to *pay* investors to choose your active fund.) These investors desire higher risk-higher return complete portfolios and thus are in the borrowing range of the relevant CAL. In this range, the reward-to-variability ratio of the index (the passive fund) is better than that of the managed fund.

- 18. b
- 19. a
- 20. a

 $(0.6 \times \$50,000) + [0.4 \times (-\$30,000)] - \$5,000 = \$13,000$

- 21. b
- 22. c

Expected return for equity fund = T-bill rate + risk premium = 6% + 10% = 16%Expected return of client's overall portfolio = $(0.6 \times 16\%) + (0.4 \times 6\%) = 12\%$ Standard deviation of client's overall portfolio = $0.6 \times 14\% = 8.4\%$

23. a

Reward to variability ratio = $\frac{10}{14} = 0.71$

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