1. a. Arithmetic average: \( \bar{r}_{ABC} = 10\% \); \( \bar{r}_{XYZ} = 10\% \)

b. Dispersion: \( \sigma_{ABC} = 7.07\% \); \( \sigma_{XYZ} = 13.91\% \)

Stock XYZ has greater dispersion.
(Note: We used 5 degrees of freedom in calculating standard deviations.)

c. Geometric average:
\[
\bar{r}_{ABC} = \left( 1.20 \times 1.12 \times 1.14 \times 1.03 \times 1.01 \right)^{1/5} - 1 = 0.0977 = 9.77\%
\]
\[
\bar{r}_{XYZ} = \left( 1.30 \times 1.12 \times 1.18 \times 1.00 \times 0.90 \right)^{1/5} - 1 = 0.0911 = 9.11\%
\]

Despite the fact that the two stocks have the same arithmetic average, the geometric average for XYZ is less than the geometric average for ABC. The reason for this result is the fact that the greater variance of XYZ drives the geometric average further below the arithmetic average.

d. In terms of “forward looking” statistics, the arithmetic average is the better estimate of expected rate of return. Therefore, if the data reflect the probabilities of future returns, 10\% is the expected rate of return for both stocks.

2. a. Time-weighted average returns are based on year-by-year rates of return:

<table>
<thead>
<tr>
<th>Year</th>
<th>Return = (capital gains + dividend)/price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998 – 1999</td>
<td>([(120 – 100) + 4]/100 = 24.00%)</td>
</tr>
<tr>
<td>1999 – 2000</td>
<td>([(90 – 120) + 4]/120 = -21.67%)</td>
</tr>
<tr>
<td>2000 – 2001</td>
<td>([(100 – 90) + 4]/90 = 15.56%)</td>
</tr>
</tbody>
</table>

Arithmetic mean: \((24\% – 21.67\% + 15.56\%)/3 = 5.96\%\)

Geometric mean: \((1.24 \times 0.7833 \times 1.1556)^{1/3} – 1 = 0.0392 = 3.92\%\)

b. 

<table>
<thead>
<tr>
<th>Date</th>
<th>Cash Flow</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/1/98</td>
<td>-$300</td>
<td>Purchase of three shares at $100 each</td>
</tr>
<tr>
<td>1/1/99</td>
<td>-$228</td>
<td>Purchase of two shares at $120 less dividend income on three shares held</td>
</tr>
<tr>
<td>1/1/00</td>
<td>$110</td>
<td>Dividends on five shares plus sale of one share at $90</td>
</tr>
<tr>
<td>1/1/01</td>
<td>$416</td>
<td>Dividends on four shares plus sale of four shares at $100 each</td>
</tr>
</tbody>
</table>
Dollar-weighted return = Internal rate of return = –0.1607%

3.

<table>
<thead>
<tr>
<th>Time</th>
<th>Cash flow</th>
<th>Holding period return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3(–$90) = −$270</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$100</td>
<td>(100–90)/90 = 11.11%</td>
</tr>
<tr>
<td>2</td>
<td>$100</td>
<td>0%</td>
</tr>
<tr>
<td>3</td>
<td>$100</td>
<td>0%</td>
</tr>
</tbody>
</table>

a. Time-weighted geometric average rate of return =

\[(1.1111 \times 1.0 \times 1.0)^{1/3} - 1 = 0.0357 = 3.57%\]

b. Time-weighted arithmetic average rate of return = \( (11.11% + 0 + 0)/3 = 3.70% \)

The arithmetic average is always greater than or equal to the geometric average; the greater the dispersion, the greater the difference.

c. Dollar-weighted average rate of return = IRR = 5.46%

[Using a financial calculator, enter: n = 3, PV = −270, FV = 0, PMT = 100. Then compute the interest rate.] The IRR exceeds the other averages because the investment fund was the largest when the highest return occurred.
4. a. The alphas for the two portfolios are:

\[ \alpha_A = 12\% - [5\% + 0.7(13\% - 5\%)] = 1.4\% \]

\[ \alpha_B = 16\% - [5\% + 1.4(13\% - 5\%)] = -0.2\% \]

Ideally, you would want to take a long position in Portfolio A and a short position in Portfolio B.

b. If you will hold only one of the two portfolios, then the Sharpe measure is the appropriate criterion:

\[ S_A = \frac{12 - 5}{12} = 0.583 \]

\[ S_B = \frac{16 - 5}{31} = 0.355 \]

Using the Sharpe criterion, Portfolio A is the preferred portfolio.

5. a. | Stock A | Stock B |
--- | --- | --- |
(i) Alpha = regression intercept | 1.0% | 2.0% |
(ii) Information ratio = \( \frac{\alpha_P}{\sigma(e_P)} \) | 0.0971 | 0.1047 |
(iii) *Sharpe measure = \( \frac{r_P - r_f}{\sigma_P} \) | 0.4907 | 0.3373 |
(iv) **Treynor measure = \( \frac{r_P - r_f}{\beta_P} \) | 8.833 | 10.500 |

* To compute the Sharpe measure, note that for each stock, \( r_P - r_f \) can be computed from the right-hand side of the regression equation, using the assumed parameters \( r_M = 14\% \) and \( r_f = 6\% \). The standard deviation of each stock’s returns is given in the problem.

** The beta to use for the Treynor measure is the slope coefficient of the regression equation presented in the problem.

b. (i) If this is the only risky asset held by the investor, then Sharpe’s measure is the appropriate measure. Since the Sharpe measure is higher for Stock A, then A is the best choice.

(ii) If the stock is mixed with the market index fund, then the contribution to the overall Sharpe measure is determined by the appraisal ratio; therefore, Stock B is preferred.

(iii) If the stock is one of many stocks, then Treynor’s measure is the appropriate measure, and Stock B is preferred.
6. We need to distinguish between market timing and security selection abilities. The intercept of the scatter diagram is a measure of stock selection ability. If the manager tends to have a positive excess return even when the market’s performance is merely “neutral” (i.e., has zero excess return), then we conclude that the manager has on average made good stock picks. Stock selection must be the source of the positive excess returns.

Timing ability is indicated by the curvature of the plotted line. Lines that become steeper as you move to the right along the horizontal axis show good timing ability. The steeper slope shows that the manager maintained higher portfolio sensitivity to market swings (i.e., a higher beta) in periods when the market performed well. This ability to choose more market-sensitive securities in anticipation of market upturns is the essence of good timing. In contrast, a declining slope as you move to the right means that the portfolio was more sensitive to the market when the market did poorly and less sensitive when the market did well. This indicates poor timing.

We can therefore classify performance for the four managers as follows:

<table>
<thead>
<tr>
<th>Selection Ability</th>
<th>Timing Ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Bad</td>
<td>Good</td>
</tr>
<tr>
<td>B. Good</td>
<td>Good</td>
</tr>
<tr>
<td>C. Good</td>
<td>Bad</td>
</tr>
<tr>
<td>D. Bad</td>
<td>Bad</td>
</tr>
</tbody>
</table>

7. a. Bogey: \((0.60 \times 2.5\%) + (0.30 \times 1.2\%) + (0.10 \times 0.5\%) = 1.91\%\)

Actual: \((0.70 \times 2.0\%) + (0.20 \times 1.0\%) + (0.10 \times 0.5\%) = 1.65\%\)

Underperformance: 0.26%

b. Security Selection:

<table>
<thead>
<tr>
<th>Market</th>
<th>Differential return within market (Manager – index)</th>
<th>Manager’s portfolio weight</th>
<th>Contribution to performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>–0.5%</td>
<td>0.70</td>
<td>–0.35%</td>
</tr>
<tr>
<td>Bonds</td>
<td>–0.2%</td>
<td>0.20</td>
<td>–0.04%</td>
</tr>
<tr>
<td>Cash</td>
<td>0.0%</td>
<td>0.10</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Contribution of security selection: –0.39%
c. Asset Allocation:

<table>
<thead>
<tr>
<th>Market</th>
<th>(1) Excess weight (Manager – benchmark)</th>
<th>(2) Index Return</th>
<th>(3) = (1) \times (2) Contribution to performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.10%</td>
<td>2.5%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Bonds</td>
<td>−0.10%</td>
<td>1.2%</td>
<td>−0.12%</td>
</tr>
<tr>
<td>Cash</td>
<td>0.00%</td>
<td>0.5%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Contribution of asset allocation: 0.13%

Summary:
Security selection −0.39%
Asset allocation 0.13%
Excess performance −0.26%

8. a. Manager return: \((0.30 \times 20) + (0.10 \times 15) + (0.40 \times 10) + (0.20 \times 5) = 12.50\%\)
Benchmark (bogey): \((0.15 \times 12) + (0.30 \times 15) + (0.45 \times 14) + (0.10 \times 12) = 13.80\%\)

Added value: −1.30%

b. Added value from country allocation:

<table>
<thead>
<tr>
<th>Country</th>
<th>(1) Excess weight (Manager – benchmark)</th>
<th>(2) Index Return minus bogey</th>
<th>(3) = (1) \times (2) Contribution to performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>0.15%</td>
<td>−1.8%</td>
<td>−0.27%</td>
</tr>
<tr>
<td>Japan</td>
<td>−0.20%</td>
<td>1.2%</td>
<td>−0.24%</td>
</tr>
<tr>
<td>U.S.</td>
<td>−0.05%</td>
<td>0.2%</td>
<td>−0.01%</td>
</tr>
<tr>
<td>Germany</td>
<td>0.10%</td>
<td>−1.8%</td>
<td>−0.18%</td>
</tr>
</tbody>
</table>

Contribution of country allocation: −0.70%

c. Added value from stock selection:

<table>
<thead>
<tr>
<th>Country</th>
<th>(1) Differential return within country (Manager – Index)</th>
<th>(2) Manager’s country weight</th>
<th>(3) = (1) \times (2) Contribution to performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.K.</td>
<td>8%</td>
<td>0.30%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Japan</td>
<td>0%</td>
<td>0.10%</td>
<td>0.0%</td>
</tr>
<tr>
<td>U.S.</td>
<td>−4%</td>
<td>0.40%</td>
<td>−1.6%</td>
</tr>
<tr>
<td>Germany</td>
<td>−7%</td>
<td>0.20%</td>
<td>−1.4%</td>
</tr>
</tbody>
</table>

Contribution of stock selection: −0.6%

Summary:
Country allocation −0.70%
Stock selection −0.60%
Excess performance −1.30%
9. **Support:** A manager could be a better performer in one type of circumstance than in another. For example, a manager who does no timing, but simply maintains a high beta, will do better in up markets and worse in down markets. Therefore, we should observe performance over an entire cycle. Also, to the extent that observing a manager over an entire cycle increases the number of observations, it would improve the reliability of the measurement.

**Contradict:** If we adequately control for exposure to the market (i.e., adjust for beta), then market performance should not affect the relative performance of individual managers. It is therefore not necessary to wait for an entire market cycle to pass before evaluating a manager.

10. The use of universes of managers to evaluate relative investment performance does, to some extent, overcome statistical problems, as long as those manager groups can be made sufficiently homogeneous with respect to style.

11. a. The manager’s alpha is: $10\% - [6\% + 0.5(14\% - 6\%)] = 0$

   b. From Black-Jensen-Scholes and others, we know that, on average, portfolios with low beta have historically had positive alphas. (The slope of the empirical security market line is shallower than predicted by the CAPM.) Therefore, given the manager’s low beta, performance might actually be sub-par despite the estimated alpha of zero.

12. a. **Manager A**

   **Strength.** Although Manager A’s one-year total return was somewhat below the international index return (–6.0 percent versus –5.0 percent), this manager apparently has some country/security return expertise. This large local market return advantage of 2.0 percent exceeds the 0.2 percent return for the international index.

   **Weakness.** Manager A has an obvious weakness in the currency management area. This manager experienced a marked currency return shortfall, with a return of –8.0 percent versus –5.2 percent for the index.

   **Manager B**

   **Strength.** Manager B’s total return exceeded that of the index, with a marked positive increment apparent in the currency return. Manager B had a –1.0 percent currency return compared to a –5.2 percent currency return on the international index. Based on this outcome, Manager B’s strength appears to be expertise in the currency selection area.

   **Weakness.** Manager B had a marked shortfall in local market return. Therefore, Manager B appears to be weak in security-market selection ability.
b. The following strategies would enable the fund to take advantage of the strengths of each of the two managers while minimizing their weaknesses.

1. **Recommendation:** One strategy would be to direct Manager A to make no currency bets relative to the international index and to direct Manager B to make only currency decisions, and no active country or security selection bets.
   
   **Justification:** This strategy would mitigate Manager A’s weakness by hedging all currency exposures into index-like weights. This would allow capture of Manager A’s country and stock selection skills while avoiding losses from poor currency management. This strategy would also mitigate Manager B’s weakness, leaving an index-like portfolio construct and capitalizing on the apparent skill in currency management.

2. **Recommendation:** Another strategy would be to combine the portfolios of Manager A and Manager B, with Manager A making country exposure and security selection decisions and Manager B managing the currency exposures created by Manager A’s decisions (providing a “currency overlay”).
   
   **Justification:** This recommendation would capture the strengths of both Manager A and Manager B and would minimize their collective weaknesses.

13. a. Indeed, the one year results were terrible, but one year is a poor statistical base from which to draw inferences. Moreover, the board of trustees had directed Karl to adopt a long-term horizon. The Board specifically instructed the investment manager to give priority to long term results.

b. The sample of pension funds had a much larger share invested in equities than did Alpine. Equities performed much better than bonds. Yet the trustees told Alpine to hold down risk, investing not more than 25% of the plan’s assets in common stocks. (Alpine’s beta was also somewhat defensive.) Alpine should not be held responsible for an asset allocation policy dictated by the client.

c. Alpine’s alpha measures its risk-adjusted performance compared to the market:

   \[ \alpha = 13.3\% - [7.5\% + 0.90(13.8\% - 7.5\%)] = 0.13\% \] (actually above zero)

d. Note that the last 5 years, and particularly the most recent year, have been bad for bonds, the asset class that Alpine had been encouraged to hold. Within this asset class, however, Alpine did much better than the index fund. Moreover, despite the fact that the bond index underperformed both the actuarial return and T-bills, Alpine outperformed both. Alpine’s performance within each asset class has been superior on a risk-adjusted basis. Its overall disappointing returns were due to a heavy asset allocation weighting towards bonds, which was the Board’s, not Alpine’s, choice.
e. A trustee may not care about the time-weighted return, but that return is more indicative of the manager’s performance. After all, the manager has no control over the cash inflows and outflows of the fund.

14. a. Method I does nothing to separately identify the effects of market timing and security selection decisions. It also uses a questionable “neutral position,” the composition of the portfolio at the beginning of the year.

b. Method II is not perfect, but is the best of the three techniques. It at least attempts to focus on market timing by examining the returns for portfolios constructed from bond market indexes using actual weights in various indexes versus year-average weights. The problem with this method is that the year-average weights need not correspond to a client’s “neutral” weights. For example, what if the manager were optimistic over the entire year regarding long-term bonds? Her average weighting could reflect her optimism, and not a neutral position.

c. Method III uses net purchases of bonds as a signal of bond manager optimism. But such net purchases can be motivated by withdrawals from or contributions to the fund rather than the manager’s decisions. (Note that this is an open-ended mutual fund.) Therefore, it is inappropriate to evaluate the manager based on whether net purchases turn out to be reliable bullish or bearish signals.

15. Treynor measure = \((17 - 8)/1.1 = 8.182\)

16. Sharpe measure = \((24 - 8)/18 = 0.888\)

17. a. Treynor measures
   Portfolio X: \((10 - 6)/0.6 = 6.67\)
   S&P 500: \((12 - 6)/1.0 = 6.00\)

   Sharpe measures
   Portfolio X: \((10 - 6)/18 = 0.222\)
   S&P 500: \((12 - 6)/13 = 0.462\)

   Portfolio X outperforms the market based on the Treynor measure, but underperforms based on the Sharpe measure.

b. The two measures of performance are in conflict because they use different measures of risk. Portfolio X has less systematic risk than the market, as measured by its lower beta, but more total risk (volatility), as measured by its higher standard deviation. Therefore, the portfolio outperforms the market based on the Treynor measure but underperforms based on the Sharpe measure.
18. Geometric average = $(1.15 \times 0.90)^{1/2} - 1 = 0.0173 = 1.73\%$

19. Geometric average = $(0.91 \times 1.23 \times 1.17)^{1/3} - 1 = 0.0941 = 9.41\%$

20. Internal rate of return = 7.5\%

21. d.

22. Time-weighted average return = $(15\% + 10\%)/2 = 12.5\%$

To compute dollar-weighted rate of return, cash flows are:

\[
\begin{align*}
CF_0 &= -$500,000 \\
CF_1 &= -$500,000 \\
CF_2 &= ($500,000 \times 1.15 \times 1.10) + ($500,000 \times 1.10) = $1,182,500
\end{align*}
\]

Dollar-weighted rate of return = 11.71\%

23. b.

24. a.

25. a. Each of these benchmarks has several deficiencies, as described below.

Market index:

- A market index may exhibit survivorship bias. Firms that have gone out of business are removed from the index, resulting in a performance measure that overstates actual performance had the failed firms been included.
- A market index may exhibit double counting that arises because of companies owning other companies and both being represented in the index.
- It is often difficult to exactly and continually replicate the holdings in the market index without incurring substantial trading costs.
- The chosen index may not be an appropriate proxy for the management style of the managers.
- The chosen index may not represent the entire universe of securities. For example, the S&P 500 Index represents 65\% to 70\% of U.S. equity market capitalization.
- The chosen index (e.g., the S&P 500) may have a large capitalization bias.
- The chosen index may not be investable. There may be securities in the index that cannot be held in the portfolio.
Benchmark normal portfolio:
• This is the most difficult performance measurement method to develop and calculate.
• The normal portfolio must be continually updated, requiring substantial resources.
• Consultants and clients are concerned that managers who are involved in developing and calculating their benchmark portfolio may produce an easily-beaten normal portfolio, making their performance appear better than it actually is.

Median of the manager universe:
• It can be difficult to identify a universe of managers appropriate for the investment style of the plan’s managers.
• Selection of a manager universe for comparison involves some, perhaps much, subjective judgement.
• Comparison with a manager universe does not take into account the risk taken in the portfolio.
• The median of a manager universe does not represent an “investable” portfolio; that is, a portfolio manager may not be able to invest in the median manager portfolio.
• Such a benchmark may be ambiguous. The names and weights of the securities constituting the benchmark are not clearly delineated.
• The benchmark is not constructed prior to the start of an evaluation period; it is not specified in advance.
• A manager universe may exhibit survivorship bias; managers who have gone out of business are removed from the universe, resulting in a performance measure that overstates the actual performance had those managers been included.

b. i. The Sharpe ratio is calculated by dividing the portfolio risk premium (i.e., actual portfolio return minus the risk-free return) by the portfolio standard deviation:

\[ \text{Sharpe ratio} = \frac{(r_P - r_f)}{\sigma_P} \]

The Treynor measure is calculated by dividing the portfolio risk premium (i.e., actual portfolio return minus the risk-free return) by the portfolio beta:

\[ \text{Treynor measure} = \frac{(r_P - r_f)}{\beta_P} \]

Jensen’s alpha is calculated by subtracting the market risk premium, adjusted for risk by the portfolio’s beta, from the actual portfolio excess return (risk premium). It can be described as the difference in return earned by the portfolio compared to the return implied by the Capital Asset Pricing Model or Security Market Line:

\[ \alpha_p = r_P - [r_f + \beta_P (r_M - r_f)] \]
ii. The Sharpe ratio assumes that the relevant risk is total risk, and it measures excess return per unit of total risk. The Treynor measure assumes that the relevant risk is systematic risk, and it measures excess return per unit of systematic risk. Jensen’s alpha assumes that the relevant risk is systematic risk, and it measures excess return at a given level of systematic risk.

26. i. The statement is incorrect. Valid benchmarks are unbiased. Median manager benchmarks, however, are subject to significant survivorship bias, which results in several drawbacks, including the following:
  • The performance of median manager benchmarks is biased upwards.
  • The upward bias increases with time.
  • Survivor bias introduces uncertainty with regard to manager rankings.
  • Survivor bias skews the shape of the distribution curve.

ii. The statement is incorrect. Valid benchmarks are unambiguous and able to be replicated. The median manager benchmark, however, is ambiguous because the weights of the individual securities in the benchmark are not known. The portfolio’s composition cannot be known before the conclusion of a measurement period because identification as a median manager can occur only after performance is measured.

Valid benchmarks are also investable. The median manager benchmark is not investable. That is, a manager using a median manager benchmark cannot forego active management and, taking a passive/indexed approach, simply hold the benchmark. This is a result of the fact that the weights of individual securities in the benchmark are not known.

iii. The statement is correct. The median manager benchmark may be inappropriate because the median manager universe encompasses many investment styles and, therefore, may not be consistent with a given manager’s style.

27. a. Sharpe ratio = \( \frac{r_P - r_f}{\sigma_P} \)

Williamson Capital: Sharpe ratio = \( \frac{22.1\% - 5.0\%}{16.8\%} = 1.02 \)
Joyner Asset Management: Sharpe ratio = \( \frac{24.2\% - 5.0\%}{20.2\%} = 0.95 \)

Treynor measure = \( \frac{r_P - r_f}{\beta_P} \)

Williamson Capital: Treynor measure = \( \frac{22.1\% - 5.0\%}{1.2} = 14.25 \)
Joyner Asset Management: Treynor measure = \( \frac{24.2\% - 5.0\%}{0.8} = 24.00 \)

b. The difference in the rankings of Williamson and Joyner results directly from the difference in diversification of the portfolios. Joyner has a higher Treynor measure (24.00) and a lower Sharpe ratio (0.95) than does Williamson (14.25 and 1.202, respectively), so Joyner must be less diversified than Williamson. The Treynor measure indicates that Joyner has a higher return per unit of systematic risk than does Williamson, while the Sharpe ratio indicates that Joyner has a lower return per unit of total risk than does Williamson.