Chapter 6
Efficient Diversification

1. $E(r_P) = 12.1\%$

3.

a. The mean return should be equal to the value computed in the spreadsheet. The fund's return is 3% lower in a recession, but 3% higher in a boom. However, the variance of returns should be higher, reflecting the greater dispersion of outcomes in the three scenarios.

b. Calculation of mean return and variance for the stock fund:

(A)	(B)	(C)	(D)	(E)	(F)	(G)
Scenario	Probability	Rate of Return	Col. B × Col. C	Deviation from Expected Return	Squared Deviation	Col. B × Col. F
Recession	0.3	-14	-4.2	-24	576	172.8
Normal	0.4	13	5.2	3	9	3.6
Boom	0.3	30	9	20	400	120
		Expected Return =	10		Variance =	296.4
					Standard Deviation =	17.22

c. Calculation of covariance:

(A)	(B)	(C)	(D)	(E)	(F)
		Deviation from Mean Return			
Scenario	Probability	Stock Fund	Bond Fund	Col. C × Col. D	Col. B × Col. E
Recession	0.3	-24	10	-240	-72
Normal	0.4	3	0	0	0
Boom	0.3	20	-10	-200	-60
				Covariance =	-132

Covariance has increased because the stock returns are more extreme in the recession and boom periods. This makes the tendency for stock returns to be poor when bond returns are good (and vice versa) even more dramatic.

4.

a. One would expect variance to increase because the probabilities of the extreme outcomes are now higher.

b. Calculation of mean return and variance for the stock fund:

(A)	(B)	(C)	(D)	(E)	(F)	(G)
Scenario	Probability	Rate of Return	Col. B × Col. C	Deviation from Expected Return	Squared Deviation	Col. B × Col. F
Recession	0.4	-11	-4.4	-20	400	160
Normal	0.2	13	2.6	4	16	3.2
Boom	0.4	27	10.8	18	324	129.6
		Expected Return =	9		Variance =	292.8
					Standard Deviation =	17.11

c. Calculation of covariance

(A)	(B)	(C)	(D)	(E)	(F)
		Deviation from Mean Return			
Scenario	Probability	Stock Fund	Bond Fund	Col. C × Col. D	Col. B × Col. E
Recession	0.4	-20	10	-200	-80
Normal	0.2	4	0	0	0
Boom	0.4	18	-10	-180	-72
				Covariance =	-152

Covariance has increased because the probabilities of the more extreme returns in the recession and boom periods are now higher. This makes the tendency for stock returns to be poor when bond returns are good (and vice versa) more dramatic.

5.

a. Subscript OP refers to the original portfolio, ABC to the new stock, and NP to the new portfolio.

   i. $E(r_{NP}) = w_{OP}\, E(r_{OP}) + w_{ABC}\, E(r_{ABC}) = 0.728\%$

   ii. $Cov = r \times \sigma_{OP} \times \sigma_{ABC} = 2.7966 \cong 2.80$

   iii. $\sigma_{NP} = [w_{OP}^2\, \sigma_{OP}^2 + w_{ABC}^2\, \sigma_{ABC}^2 + 2\, w_{OP}\, w_{ABC}\, (Cov_{OP,\,ABC})]^{1/2}$

   $= 2.2673\% \cong 2.27\%$

b. Subscript OP refers to the original portfolio, GS to government securities, and NP to the new portfolio.

   i. $E(r_{NP}) = w_{OP}\, E(r_{OP}) + w_{GS}\, E(r_{GS}) = 0.645\%$

   ii. $Cov = r \times \sigma_{OP} \times \sigma_{GS} = 0 \times 2.37 \times 0 = 0$

   iii. $\sigma_{NP} = [w_{OP}^2\, \sigma_{OP}^2 + w_{GS}^2\, \sigma_{GS}^2 + 2\, w_{OP}\, w_{GS}\, (Cov_{OP,\,GS})]^{1/2}$

   $= 2.133\% \cong 2.13\%$

c. Adding the risk-free government securities would result in a lower beta for the new portfolio. The new portfolio beta will be a weighted average of the individual security betas in the portfolio; the presence of the risk-free securities would lower that weighted average.

d. The comment is not correct. Although the respective standard deviations and expected returns for the two securities under consideration are equal, the covariances between each security and the original portfolio are unknown, making it impossible to draw the conclusion stated. For instance, if the covariances are different, selecting one security over the other may result in a lower standard deviation for the portfolio as a whole. In such a case, that security would be the preferred investment, assuming all other factors are equal.

e. Grace clearly expressed the sentiment that the risk of loss was more important to her than the opportunity for return. Using variance (or standard deviation) as a measure of risk in her case has a serious limitation because standard deviation does not distinguish between positive and negative price movements.

6. The parameters of the opportunity set are:

$E(r_S) = 15\%$, $E(r_B) = 9\%$, $\sigma_S = 32\%$, $\sigma_B = 23\%$, $\rho = 0.15$, $r_f = 5.5\%$

From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho\sigma_S\sigma_B$]:

	Bonds	Stocks
Bonds	529.0	110.4
Stocks	110.4	1024.0

The minimum-variance portfolio proportions are:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = 0.3142$$

$w_{Min}(B) = 0.6858$

The mean and standard deviation of the minimum variance portfolio are:

$E(r_{Min}) = 10.89\%$

$$\sigma_{Min} = \left[w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)\right]^{1/2}$$

$= 19.94\%$

% in stocks	% in bonds	Exp. return	Std dev.	
00.00	100.00	9.00	23.00	
20.00	80.00	10.20	20.37	
31.42	**68.58**	**10.89**	**19.94**	**Minimum variance**
40.00	60.00	11.40	20.18	
60.00	40.00	12.60	22.50	
70.75	**29.25**	**13.25**	**24.57**	**Tangency portfolio**
80.00	20.00	13.80	26.68	
100.00	00.00	15.00	32.00	

8. The reward-to-variability ratio of the optimal CAL is: 0.3154

14. If the lending and borrowing rates are equal and there are no other constraints on portfolio choice, then optimal risky portfolios of all investors will be identical. However, if the borrowing and lending rates are not equal, then borrowers (who are relatively risk averse) and lenders (who are relatively risk tolerant) will have different optimal risky portfolios.

15. No, it is not possible to get such a diagram. Even if the correlation between A and B were 1.0, the frontier would be a straight line connecting A and B.

18. The expected rate of return on the stock will change by beta times the unanticipated change in the market return: – 2.4%

    Therefore, the expected rate of return on the stock should be revised to: 12% – 2.4% = 9.6%

22.

    a. Restricting the portfolio to 20 stocks, rather than 40 to 50, will very likely increase the risk of the portfolio, due to the reduction in diversification. Such an increase might be acceptable if the expected return is increased sufficiently.

    b. Hennessy could contain the increase in risk by making sure that he maintains reasonable diversification among the 20 stocks that remain in his portfolio. This entails maintaining a low correlation among the remaining stocks. As a practical matter, this means that Hennessy would need to spread his portfolio among many industries, rather than concentrating in just a few.

23. Risk reduction benefits from diversification are not a linear function of the number of issues in the portfolio. (See Figures 6.1 and 6.2 in the text.) Rather, the incremental benefits from additional diversification are most important when the portfolio is least diversified. Restricting Hennessy to 10 issues, instead of 20 issues, would increase the risk of his portfolio by a greater amount than reducing the size of the portfolio from 30 to 20 stocks.