

Chapter 7 Capital Asset Pricing and Arbitrage Pricing Theory

1. a, c and d

2.

a. $E(r_X) = 12.2\%$

$$\alpha_X = 1.8\%$$

$$E(r_Y) = 18.5\%$$

$$\alpha_Y = -1.5\%$$

b.

(i) For an investor who wants to add this stock to a well-diversified equity portfolio, Kay should recommend Stock X because of its positive alpha, while Stock Y has a negative alpha. In graphical terms, Stock X's expected return/risk profile plots above the SML, while Stock Y's profile plots below the SML. Also, depending on the individual risk preferences of Kay's clients, Stock X's lower beta may have a beneficial impact on overall portfolio risk.

(ii) For an investor who wants to hold this stock as a single-stock portfolio, Kay should recommend Stock Y, because it has higher forecasted return and lower standard deviation than Stock X. Stock Y's Sharpe ratio is:

$$0.48$$

Stock X's Sharpe ratio is only:

$$0.25$$

The market index has an even more attractive Sharpe ratio:

$$0.60$$

However, given the choice between Stock X and Y, Y is superior. When a stock is held in isolation, standard deviation is the relevant risk measure. For assets held in isolation, beta as a measure of risk is irrelevant. Although holding a single asset in isolation is not typically a recommended investment strategy, some investors may hold what is essentially a single-asset portfolio (e.g., the stock of their employer company). For such investors, the relevance of standard deviation versus beta is an important issue.

3. $E(r_P) = r_f + \beta[E(r_M) - r_f]$

$$\beta = 1.5$$

6.

- a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.
- b. False. Investors require a risk premium for bearing systematic (i.e., market or undiversifiable) risk.
- c. False. You should invest 0.75 of your portfolio in the market portfolio, and the remainder in T-bills. Then:

$$\beta_P = 0.75$$

7.

- a. The beta is the sensitivity of the stock's return to the market return. Call the aggressive stock *A* and the defensive stock *D*. Then beta is the change in the stock return per unit change in the market return. We compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in market return.

$$\beta_A = \frac{2 - 32}{5 - 20} = 2.00$$

$$\beta_D = \frac{3.5 - 14}{5 - 20} = 0.70$$

- b. With the two scenarios equal likely, the expected rate of return is an average of the two possible outcomes:

$$E(r_A) = 17\%$$

$$E(r_B) = 8.75\%$$

8. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower.

9. Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk as measured by beta, rather than the standard deviation, which includes nonsystematic risk. Thus, Portfolio A's lower expected rate of return can be paired with a higher standard deviation, as long as Portfolio A's beta is lower than that of Portfolio B.

10. Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market, which is not possible according to the CAPM, since the CAPM predicts that the market portfolio is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{16 - 10}{12} = 0.5$$

$$S_M = \frac{18 - 10}{24} = 0.33$$

These figures imply that Portfolio A provides a better risk-reward tradeoff than the market portfolio.

11. Not possible. Portfolio A clearly dominates the market portfolio. It has a lower standard deviation with a higher expected return.

12. Not possible. Given these data, the SML is: $E(r) = 10\% + \beta(18\% - 10\%)$

A portfolio with beta of 1.5 should have an expected return of:

$$E(r) = 22\%$$

The expected return for Portfolio A is 16% so that Portfolio A plots below the SML (i.e., has an alpha of -6%), and hence is an overpriced portfolio. This is inconsistent with the CAPM.

13. Not possible. The SML is the same as in Problem 12. Here, the required expected return for Portfolio A is: 17.2%

This is still higher than 16%. Portfolio A is overpriced, with alpha equal to: -1.2%

14. Possible. Portfolio A's ratio of risk premium to standard deviation is less attractive than the market's. This situation is consistent with the CAPM. The market portfolio should provide the highest reward-to-variability ratio.

16. Since the stock's beta is equal to 1.0, its expected rate of return should be equal to that of the market, that is, 18%.

$$E(r) = \frac{D + P_1 - P_0}{P_0}$$

$$\Rightarrow P_1 = \$109$$

17. If beta is zero, the cash flow should be discounted at the risk-free rate, 8%:

$$PV = \$12,500$$

If, however, beta is actually equal to 1, the investment should yield 18%, and the price paid for the firm should be:

$$PV = \$5,555.56$$

The difference (\$6944.44) is the amount you will overpay if you erroneously assume that beta is zero rather than 1.

18. Using the SML: $\beta = -2/10 = -0.2$

19. $r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1.0$

- a. In order to determine which investor was a better selector of individual stocks we look at the abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (i.e., the risk-free rate and the market rate of return) we cannot determine which investment adviser is the better selector of individual stocks.

If $r_f = 6\%$ and $r_M = 14\%$, then (using alpha for the abnormal return):

$$\alpha_1 = 1\%$$

$$\alpha_2 = 2\%$$

Here, the second investment adviser has the larger abnormal return and thus appears to be the better selector of individual stocks. By making better predictions, the second adviser appears to have tilted his portfolio toward under-priced stocks.

- b. If $r_f = 3\%$ and $r_M = 15\%$, then:

$$\alpha_1 = -2\%$$

$$\alpha_2 = 1\%$$

Here, not only does the second investment adviser appear to be a better stock selector, but the first adviser's selections appear valueless (or worse).

20.

- a. Since the market portfolio, by definition, has a beta of 1.0, its expected rate of return is 12%.
- b. $\beta = 0$ means the stock has no systematic risk. Hence, the portfolio's expected rate of return is the risk-free rate, 4%.
- c. Using the SML, the *fair* rate of return for a stock with $\beta = -0.5$ is:

$$E(r) = 0.0\%$$

The *expected* rate of return, using the expected price and dividend for next year:

$$E(r) = 10\%$$

Because the expected return exceeds the fair return, the stock must be under-priced.

21. The data can be summarized as follows:

	Expected Return	Beta	Deviation
Portfolio A	11%	0.8	10%
Portfolio B	14%	1.5	31%
S & P 500	12%	1	20%
T-bills	6%	0	0%

- a. Using the SML, the expected rate of return for any portfolio P is:

$$E(r_P) = r_f + \beta[E(r_M) - r_f]$$

Substituting for portfolios A and B:

$$E(r_A) = 10.8\%$$

$$E(r_B) = 15.0\%$$

Hence, Portfolio A is desirable and Portfolio B is not.

- b. The slope of the CAL supported by a portfolio P is given by:

$$S = \frac{E(r_P) - r_f}{\sigma_P}$$

Computing this slope for each of the three alternative portfolios, we have:

$$S(\text{S\&P 500}) = 6/20$$

$$S(A) = 5/10$$

$$S(B) = 8/31$$

Hence, portfolio A would be a good substitute for the S&P 500.