Solution to FIN 533 Homework
Due Tuesday September 29

1. The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{Min}(B) = 1 - 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{Min}) = (0.1739 \times 20) + (0.8261 \times 12) = 13.39\%$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$= 13.92\%$

2. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S) - r_f]\sigma_B^2 - [E(r_B) - r_f]Cov(r_S, r_B)}{[E(r_S) - r_f]\sigma_B^2 + [E(r_B) - r_f]\sigma_S^2 - [E(r_S) - r_f + E(r_B) - r_f]Cov(r_S, r_B)}$$

$$= \frac{[(20-8) \times 225] - [(12-8) \times 45]}{[(20-8) \times 225] + [(12-8) \times 900] - [(20-8+12-8) \times 45]} = 0.4516$$

$w_B = 1 - 0.4516 = 0.5484$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P) = (0.4516 \times 20) + (0.5484 \times 12) = 15.61\%$

$$\sigma_p = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2}$$

$= 16.54\%$

The reward-to-variability ratio of the optimal CAL is:

$$\frac{E(r_p) - r_f}{\sigma_p} = \frac{15.61 - 8}{16.54} = 0.4601$$

3. a. $14\% = 15.61\%\ y + (1-y)\ 8\%$ $y = .7884$

$\sigma = .7884\ (16.54\%) = 13.04\%$

b.

Proportion of stocks in complete portfolio = $0.7884 \times 0.4516 = 0.3560$

Proportion of bonds in complete portfolio = $0.7884 \times 0.5484 = 0.4324$

Proportion of T-bills in complete portfolio = $1 - 0.7884 = 0.2116$