Chapter 5
Time Value of Money

1. Suppose you deposit $100 in a bank that pays 10% interest per year. How much will you have in the bank one year later?

2. Suppose you deposit $100 in a bank that pays 10% per year. How much will you have in the bank two years later?

Future value of a single cash flow: \( FV_n = PV(1 + i)^n \)

3. Suppose you deposit $100 in a bank that pays 15% per year. How much will you have in the bank two years later?

Note: All else being the same, as the interest rate increases, the future value ____________, and as the number of periods increases, the future value ____________.

4. Suppose you deposit $100 one year from now, and $200 two years from now. How much will you have in the bank 4 years from today if the interest rate is 8% per year?

5. If a deposit of $250 today grows to $341.91 in three years, what is the annual interest rate earned?
6. If the interest rate is 6% per year, how long will it take to triple your money?

7. Find the present value of $200 to be received one year from today if the interest rate is 10% per year.

Present value of a single cash flow: \( PV = \frac{FV_n}{(1 + i)^n} = FV_n(1 + i)^{-n} \)

8. Find the present value of $200 to be received two years from today if the interest rate is 10% per year.

9. Find the present value of $200 to be received two years from today if the interest rate is 15% per year.

Note: All else being the same, as the interest rate increases, the present value ______________, and as the number of periods increases, the present value ______________.

10. Find the present value of the following cash flows, assuming an interest rate of 10% per year. 
    \( CF_2 = +$200; \ CF_3 = -$100; \ CF_4 = +$600 \)
An Annuity is a series of payments of an equal amount at fixed intervals for a specified number of periods. An ordinary annuity has payments at the end of each period.

11. If you deposit $100 per year at the end of each of the next 3 years, how much will you have in the account three years from today (i.e. just after you make the last deposit) if the bank pays 10% per year?

Future value of an ordinary annuity: 

\[ FVA_n = PMT \left[ \frac{(1 + i)^n - 1}{i} \right] \]

12. You wish to accumulate $6,000 four years from today. How much should you deposit at the end of each year if the bank pays 10% per year?

13. How much should you deposit today in order to be able to withdraw $100 at the end of each of the next three years, if the interest rate is 10% per year?
Present Value of an Ordinary Annuity:  

\[ PVA = PMT \left( \frac{1 - \frac{1}{(1 + i)^n}}{i} \right) = PMT \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]

14. Construct a loan amortization table for a $5,000 loan at 10% to be repaid in four equal end-of-year installments.

<table>
<thead>
<tr>
<th>Year</th>
<th>Beginning Balance</th>
<th>Interest</th>
<th>Payment</th>
<th>Principal Repaid</th>
<th>Ending Balance</th>
</tr>
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<td>4</td>
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</tbody>
</table>

For the loan in the above problem, what is the remaining balance after three payments have been made?

A perpetuity is a stream of equal payments expected to continue forever.

15. Find the present value of a perpetuity of $1,200 per year if the interest rate is 10% per year.

Present value of a perpetuity:  

\[ PVP = \frac{PMT}{i} \]
16. Suppose the stated interest rate is 12% per year. What is the effective annual rate if interest is compounded:

\[ EFF\% = \left(1 + \frac{i_{NOM}}{M}\right)^M - 1.0 \]

a. annually

b. semi-annually

c. monthly

d. daily

Find the future value of $1,000 one year later if interest in each case.

17. Suppose you deposit $100 every month in an account that has a stated rate of 9% per year compounded monthly. How much will you have after three years?

18. Suppose you deposit $1,200 every year in an account that has a stated rate of 9% per year compounded monthly. How much will you have after three years?
An annuity due has payments at the beginning of each period.

19. Suppose you deposit $200 at the beginning of each of the next three years. How much will you have three years from today if the interest rate is 10% per year?

Future value of an annuity due: \[ FVA_{due} = FVA_{ordinary}(1 + i) = PMT \left( \frac{(1+i)^n - 1}{i} \right)(1 + i) \]

20. Find the present value of a three year annuity with beginning-of-year cash flows of $200 each, assuming an interest rate of 10% per year.

Present value of an annuity due: \[ PVA_{due} = PVA_{ordinary}(1 + i) = PMT \left[ \frac{1 - (1+i)^{-n}}{i} \right](1 + i) \]

21. You have deposited $100,000 today in an account that pays 10% interest per year. You plan on making a round-the-world trip 4 years from today, which will cost $30,000 at that time. You also plan on making equal annual withdrawals in each of the next nine years, beginning one year from today. Find the amount of the annual withdrawal.
22. You wish to have $2.0 million on the day you retire 20 years from today. You plan on saving an equal annual amount every year to reach your goal. All savings will be invested in a stock mutual fund, which you expect will earn about 12% per year. How much should you save annually?

23. To finance the purchase of your house, you borrow $150,000 for 30 years at a stated rate of 9% per year.
   a. What is the amount of your monthly payment?
   b. What is the total interest paid on this loan?