

## Sample Problems—Time Value of Money

1. Gomez Electronics needs to arrange financing for its expansion program. Bank A offers to lend Gomez the required funds on a loan where interest must be paid monthly, and the quoted annual rate is 8 percent. Bank B will charge 9 percent, with interest due at the end of the year. What is the difference in the effective annual rates charges by the two banks?
2. In 1889, Vincent Van Gogh's painting, "Sunflowers", sold for \$125. In 1987 it sold for \$36 million. Had the painting been purchased by your great-grandfather and passed on to you, how much would your average annually compounded rate of return have been— $n=98$ .
3. You deposited \$1,000 in a savings account that pays 8 percent interest, compounded quarterly, planning to use it to finish your last year in college. Eighteen months later, you decide to go the Rocky Mountains to become a ski instructor rather than continue in school, so you close out your account. How much money will you receive?
4. Smokey Stack smokes a pack of cigarettes a day. Each pack of cigarettes costs \$2.25. Smokey, having learned about the time value of money in his finance class, is wondering how much money he could accumulate if he quit smoking and invested the money. Assuming, the **amount saved each year** is invested at the end of the year at a 8% rate of return, how much could Smokey accumulate during the next 40 years (assume a 365-day year and that the cost of cigarettes stays constant over the 40 year period)
5. Your brother-in-law borrowed \$1,000 from you 10 years ago and then disappeared. Yesterday he returned and expressed a desire to pay back the loan, including the interest that accrued since he borrowed the money. Assuming that you had agreed to charge him 7%, and assuming that he wishes to make five equal annual payments beginning in one year, how much would your brother-in-law have to pay you annually in order to extinguish the debt? (Assume the loan continues to accrue interest at 7% per year.)
6. You plan to deposit \$2,500 annually into your retirement account. You are trying to decide whether to deposit the funds at the beginning of the year or at the end of the year. How much more will you have with beginning-of-the year deposits if you deposit to the account for the next 40 years and the applicable interest rate is 10 percent?
7. Drexel Corporation has been enjoying a phenomenal rate of growth since its inception one year ago. Currently, its assets total \$100,000. If growth continues at the current annual rate of 12 percent, compounded quarterly, what will total assets be in 2 ½ years?
8. What is the effective annual percentage rate of 12 percent compounded monthly?
9. You decide to begin saving toward the purchase of a new car in 5 years. If you put \$1,000 at the beginning of each of the next 5 years in a savings account paying 6 percent compounded annually, how much will you accumulate after 5 years?
10. Steaks Galore needs to arrange financing for its expansion program. One bank offers to lend the required \$1 million on a loan with a quoted annual rate of 10 percent, compounded quarterly. A second lender offers 9 percent with daily compounding. Assume that Steaks Galore will pay both interest and principal at the end of one year. What is the difference in the effective annual rates charged by the two banks?
11. If \$100 is placed in an account that earns a simple 4 percent, compounded quarterly, what will it be worth in 5 years?
12. Seebee makes quarterly (end of period) payments of \$30,000 into a pension fund earning 12 percent per year compounded quarterly for 10 years. How much *interest* will they have earned in 10 years?

13. Mr. Moore is 35 years old today and is beginning to plan for his retirement. He wants to set aside an equal amount at the *end* of each of the next 25 years so that he can retire at age 60. He expects to live to the maximum age of 80 and wants to be able to withdraw \$25,000 per year from the account on his 61<sup>st</sup> through 80<sup>th</sup> birthdays (20 withdrawals). The account is expected to earn 10 percent per year for the entire period of time. Determine the size of the annual deposits that must be made by Mr. Moore.
14. You are considering the purchase of 20 acres of land. An analysis indicates that if the land is used for cattle grazing, it will produce a cash flow of \$1,000 per year indefinitely. If the investor requires a return of 10 percent on investments of this type, what is the most he or she should be willing to pay for this land?
15. If a 5-year ordinary annuity has a present value of \$1,000, and if the interest rate is 10 percent, what is the amount of each annuity payment.
16. If \$100 is placed in an account that earns an annual rate of 4 percent, compounded quarterly, what will it be worth in 5 years?
17. Many IRA funds argue that investors should invest at the beginning of the year rather than at the end. What is the difference to an investor who invests \$2,000 per year at 11 percent over a 30 year period?
18. Smokey Stack smokes a pack of cigarettes a day. Each pack of cigarettes costs \$3.00. Smokey, having learned about the time value of money in his finance class, is wondering how much money he could accumulate if he quit smoking and invested the money. Assuming, the **amount saved each year** is invested at the end of the year at a 9% rate of return, how much could Smokey accumulate during the next 40 years (assume a 365-day year and that the cost of cigarettes stays constant over the 40 year period)
19. A friend promises to pay you \$600 two years from now if you loan him \$500 today. What annual interest is your friend offering?
20. You are now 25 years old and planning for retirement. You have determined that you will need \$150,000 per year during your retirement. Additionally, you predict that you will live for 20 years after retirement. You plan to make annual deposits into your retirement account for the next 40 years. If the interest rate is a constant 12% during this 60-year period, how much must you deposit annually into your account?
21. The value of an asset today is \$5,000. If it earns 8% per year, how many years will it take to be worth \$30,000?
22. In six years, your daughter will be going to college. You wish to have a fund that will provide her \$10,000 per year (end-of-year) for each of her four years in college. How much must you put into that fund today if the fund will earn 10 percent in each of the 10 years?
23. You have \$500 in a savings account which earns 6% interest, compounded annually. How much additional interest would you earn in 3 years if you moved the money into an account which earns a stated 6% annual rate, compounded semiannually?
24. Suppose you are considering borrowing \$120,000 to finance your dream house. The annual percentage rate is 14.75% and payments are made monthly. If the mortgage has a 30-year amortization schedule, what are the monthly payments?
25. What is the total present value of \$50 received in one year, \$200 received in two years, \$700 received in four years, and \$800 received in six years if the discount rate is 8.25%?

26. You deposit your savings at the Darlington National Bank, which offers to pay 12.6 percent annual interest compounded monthly, or at the Bartlett Bank, which will pay annual interest of 11.5 percent compounded daily. (Assume 365 days in a year.) Which bank offers the higher effective annual rate?
27. You work for a furniture store. You normally sell a living room set for \$2,500 and finance the full purchase price for 36 monthly payments at a 22% stated annual interest. You are planning to run a zero-interest sale during which you will finance the set over 36 months at 0% interest. How much do you need to charge for the living room set during the sale in order to get the same monthly payment (as without the sale)?
28. A baseball player is offered a 5-year contract which pays him the following amounts:
- |         |               |
|---------|---------------|
| Year 1: | \$1.2 million |
| Year 2: | 1.6 million   |
| Year 3: | 2.0 million   |
| Year 4: | 2.4 million   |
| Year 5: | 2.8 million   |

Under the terms of the agreement all payments are made at the end of each year.

Instead of accepting the contract, the baseball player asks his agent to negotiate a contract which has a present value of \$1 million more than that which has been offered. Moreover, the player wants to receive his payments in the form of a 5-year annuity due. The applicable interest rate for all cash flows is 10 percent. If the team were to agree to the player's terms, what would be the player's annual salary (in millions of dollars)?

29. You just graduated, and you plan to work for 10 years and then to leave for the Australian "Outback" bush country. You figure you can save \$1,000 per year for the first 5 years and \$2,000 per year for the next 5 year. These savings cash flows will start one year from now. In addition, your family has just given you a \$5,000 graduation gift. If you put the gift now, and your future savings when they start, into an account which pays 8 percent compounded annually, what will your financial "stake" be when you leave for Australia 10 years from now?
30. Andy promises Opie that he will give Opie \$10,000 upon his graduation from college at Mayberry U. How much must Andy invest today to make good on his promise, if Opie is expected to graduate in 16 years, and Andy can earn 5% on his money?
31. In 1889, Vincent Van Gogh's painting, 'Sunflowers' sold for \$125. One hundred years later it sold for \$36 million. Had the painting been purchased by your great-grandfather and passed on to you, how much would your average annually compounded rate of return have been?
32. Ward Cleaver plans to retire 30 years from today. He would like to maintain his current standard of living, which requires an annual income of \$50,000. Given an anticipated annual rate of inflation of 3%, how much money will Ward need in his first year of retirement in order to live as he does today of \$50,000?
33. Ward Cleaver plans to retire in 30 years. He has \$12,000 to invest for his retirement. **Also**, he plans to begin depositing \$300 each month at the end of this month. If he continues to invest \$300 per month until he retires, how much will he have accumulated by the time he retires? Assume he can earn an annual rate of 8%, compounded monthly.
34. You have just won the lottery and will receive \$1,500 per year forever, with the first payment to be received in one year. What is the present value of this stream of cash flows if the interest rate is 8%?

35. Your parents agree to pay half of the purchase price of a new car when you graduate from college. You will graduate and buy the car 2 years from now. You have \$6,000 to invest today and can earn 10% on invested funds. If your parents match the amount of money you have in two years, what is the maximum you will be able to spend on the new car?

*Use the following information to answer the next three questions*

You and your spouse have found your dream home in Blackjack, MO. The price is \$100,000. You can finance your home for 30-years at a 7.25% annual rate of interest.

36. Assume that monthly payments begin in one month. What will each payment be?
37. How much interest will you pay (in dollars) over the life of the loan?
38. Although you get a 30-year mortgage, you plan to prepay the loan by making an additional payment each month along with your regular payment. How much *extra* must you pay each month if you wish to pay off the loan in 20 years?
39. You just won the lottery and want to put some money away for your child's college education. The cost for college will total \$65,000 in 18 years. You can earn 8% compounded annually. How much do you need to invest today?
40. What is the total present value of \$50 received in one year, \$200 received in two years, and \$800 received in six years if the discount rate is 8.00%?
41. You need \$2,000 to buy a new stereo for your car. If you have \$500 to invest at 14% compounded annually, how long will you have to wait to buy the stereo?
42. You have \$500 in a savings account that earns 5% compounded annually. How much additional interest would you earn over the next 4 years if you moved the money to an account that earns 6%?
43. What is the future value in 10 years of \$1,000 payments received at the beginning of each year for the next 10 years? Assume an interest rate of 5.625%?
44. Assume a new Suburban costs \$35,000 today and that you expect a 3% annual increase in the price of automobiles for the foreseeable future. How much must you set aside today to buy a new Suburban 10 years hence, if you can earn 8% annually on your money?
45. What is the future value of \$250 received today if it is invested at 6.5% compounded monthly for eight years?
46. Fast Eddie's Used Cars will sell you a 1986 Ford Escort for \$3,000 with no money down. You agree to make weekly payments for 2 years, beginning one week after you buy the car. The stated annual rate on the loan is 20%. How much is each payment?
47. Upon retirement, your goal is to spend 5 years traveling around the world. To travel in the style to which you are accustomed will require \$250,000 per year at the beginning of each year. If you plan to retire in 30 years, what are the equal, annual, end-of-year payments necessary to achieve this goal? The funds in the retirement account will compound at 10 percent annually.
48. Your child will go to college in 12 years from now and will require \$20,000, \$21,000, \$22,000 and \$23,000 at the beginning of each year in school. In addition, you and your spouse plan to retire in 20 years. You want to have \$75,000 available for each of your expected 15 years of blissful retirement. These funds will need to be available at the beginning of each year. If you have \$15,000 that can be used to meet these obligations, how much must you invest at the end of each of the next 20 years (if all funds earn an 11 percent rate of return) in order to meet your financial objectives?

49. You are 30 years old and plan to retire on your 60<sup>th</sup> birthday. You want to start a retirement plan that will require a series of equal, annual, end-of-year deposits into an account that will earn 12 percent annually. The first deposit will be made on your 31<sup>st</sup> birthday and the last payment will be on your 60<sup>th</sup> birthday. The plan will allow you to withdraw \$120,000 per year for 15 years, beginning on your 61<sup>st</sup> birthday. At the end of the 15<sup>th</sup> year, you want to withdraw an additional \$250,000. What annual payment must be made into the account to reach your objectives?
50. Many IRA funds argue that investors should invest at the beginning of the year rather than at the end. What is the difference to an investor (how **much more** will she accumulate for retirement) if she invests \$2,000 per year at 11 percent over a 30-year period?

*Use the following information to answer the next three questions:*

You have just graduated from K-State with a student loan obligation of \$5,000. The stated annual interest rate on your student loan is eight percent (compounded monthly) and you will make monthly payments for 10 years.

51. Assume that monthly payments begin at the end of this month. What will each payment be?
52. How much interest will you pay (in dollars) over the life of the loan?
53. Although the term of your student loan is 10 years, you plan to prepay the loan by making an additional payment each month along with your regular payment. How much **extra** must you pay each month if you wish to pay off the loan in 5 years?
54. Hot Wheels cars were sold at auction in 1995 by Christie's International PLC for \$100 each. At the time, it was estimated that this represented a 27.7 percent annual rate of return. For this to be true, what must the cars have sold for new in 1975?
55. Assume that you can invest to earn a stated annual 12 percent rate of return, but where interest is compounded semiannually. If you make 20 consecutive semiannual deposits of \$500 each, with the first deposit being made today, what will your balance be at the end of **year 20**?
56. An investment offers you a 12 percent annual interest rate compounded quarterly. Determine how much **additional interest** you will earn over the next 4 years on a \$10,000 investment if it is compounded quarterly compared with one that is compounded annually.
57. In 20 years you are planning on retiring and buying a house in Oviedo, Florida. The house currently costs \$300,000 and is expected to increase in value each year at a rate of 5 percent. Assuming you can earn 10 percent annually on your investments, how much must you invest at the end of each of the next 10 years to be able to buy your dream home when you retire?
58. What is the present value of receiving \$10,000 per year at the end of years 5 through 10 if you assume an interest rate of 8 percent?
59. You've just joined the investment banking firm of Peng, Yi and Lee. They've offered you two different salary arrangements. You can have \$6,250 per month for the next two years, or you can have \$4,600 per month for the next two years, along with a \$30,000 signing bonus today. If the interest rate is eight percent per year (compounded monthly), which would you prefer?
60. Your grandfather placed \$5,000 in a trust fund for you. In 12 years the fund will be worth \$10,000. What is the rate of return on the trust fund?

61. The monthly mortgage payment on your house is \$821.69. It is a 30 year mortgage at a annual 6.5% compounded monthly. How much did you borrow?
62. The Lancer Leasing Company has agreed to lease a hydraulic trencher to the Chavez Excavation Company for \$20,000 a year over the next 8 years. Lease payments are to be made at the beginning of each year. Assuming that Lancer invests these payments at an annual rate of 9 percent, how much will it have accumulated by the end of the eighth year?
63. Mr. Jones bought a building for \$50,000. He is going to finance this purchase with 25 annual payments at an interest rate of 10 percent. How much of the first year's payment goes toward reducing the principal amount?
64. Thirty years ago, Jesse Jones bought 10 acres of land for \$1,000 per acre in what is now downtown Houston. If this land grew in value at an 8 percent annual rate, what is it worth today?
65. An investment promises to pay \$6,000 at the end of each year for the next 5 years and \$4,000 at the end of each year for years 6 through 10. If you require a 12 percent rate of return on an investment of this sort, what is the maximum amount you would pay for this investment?
66. Your aunt would like to help you set up your new medical practice when you complete your medical training in 6 years. She wishes to have \$250,000 available for your use at that time. How much must she invest in an account at the end of each of the next 6 years in order to reach her goal, if the account offers a 12 percent annual rate of return?
67. The Perpetual Life Insurance Co. is trying to sell you an investment policy that will pay you and your heirs \$5,000 per year forever. If the required return on this investment is 9 percent, how much will you pay for this policy?
68. Your uncle puts \$25,000 into a bank account earning 6%. He is not going to withdraw the money until the balance has doubled. How long will he have to leave the money in the account?
- Your great-uncle Claude is 82 years old. Over the years, he has accumulated saving of \$80,000. He estimates that he will live another 10 years at the most and wants to spend his savings by then. (If he lives longer than that, he figures you will be happy to take care of him.)
69. Uncle Claude places his \$80,000 into an account earning 10 percent annually and sets it up in such a way that he will be making 10 equal annual withdrawals—the first one occurring 1 year from now—such that the balance will be zero at the end of 10 years. How much will he be able to withdraw each year?
70. First National Bank charges 9.1 percent compounded monthly on its business loans. First United Bank charges 9.2 percent compounded semiannually. As a potential borrower, which bank would you go to for a new loan? (You must substantiate your choice mathematically.)
71. Your mother is planning to retire this year. Her firm has offered her a lump sum retirement payment of \$50,000 or a \$6,000 lifetime annuity—whichever she chooses. Your mother is in reasonably good health and expects to live for at least 15 more years. Which option should she choose (which has a higher value), assuming that an 8 percent interest rate is appropriate to evaluate the annuity?

72. You are currently 30 years of age. You intend to retire at age 60 and you want to be able to receive a 20-year, \$100,000 beginning of year annuity with the first payment to be received on your sixtieth birthday. You would like to save enough money over the next 15 years to achieve your objective; that is, you want to accumulate the necessary funds by your forty-fifth birthday.

If you expect your investments to earn 10 percent, how much must you save at the end of each of the next 15 years to achieve your goal?

73. Which of the following has the highest value if you can earn a 9 percent rate of return on your investments?
- \$5,000 today
  - \$15,000 five years from today
  - \$1,000 per year for 15 years
74. Assume the total cost of a college education will be \$300,000 when your child enters college 18 years. You presently have \$40,000 to invest. What annual rate of interest must you earn on your investment to cover the cost of your child's college education?
75. You have decided that you want to be a millionaire when you retire in 40 years. If you can earn a 12% annual return, compounded monthly, how much do you have to invest today?
76. You want to buy a new sports car from Muscle Motors for \$52,350 and the finance office at the dealership has quoted you an 8.6% annual rate on the loan. You plan to pay for the car in 60 equal monthly installments. What will be your **monthly** payment?
77. You are considering an investment with a quoted return of 10% per year. If interest is compounded daily, what is the effective return on this investment?
78. What is the present value of the following set of cash flows at a 10% discount rate?
- | Year      | 1     | 2      | 3     | 4      |
|-----------|-------|--------|-------|--------|
| Cash flow | \$800 | -\$800 | \$800 | -\$800 |
79. What is the present value of \$920 per year, at a discount rate of 10 percent, if the first payment is received 5 years from now and the last payment is received 20 years from now?
80. The company you work for will deposit \$2,000 at the end of each year into your retirement fund. You plan to retire 25 years from now and estimate that you will need \$50,000 per year out of the account for the ensuing 20 years. If the account pays 10 percent annually, how much do you need to put into the account each year, in addition to your company's deposit, in order to meet your objective?
81. Homer promises Bart that he will give him \$8,000 upon his graduation from college at Springfield U. How much must Homer invest today to make good on his promise, if Bart is expected to graduate in 13 years and Homer can earn 6% on his money?

82. Your parents agree to pay half of the purchase price of a new car when you graduate from college. You will graduate and buy the car 2 years from now. You have \$6,000 to invest today and can earn 10% on invested funds. If your parents match the amount of money you have in two years, what is the maximum you will be able to spend on the new car?

83. What is the effective annual rate of 8% compounded quarterly?

84. What is the present value of the following set of end-of-year cash flows? Assume a discount rate of 8%.

Year	Cash flow
1	\$1,000
2	-\$1,000
3	\$1,000
4	-\$1,000

85. You need \$3,000 to buy a new home stereo. If you have \$1,200 to invest at 6% compounded annually, how long will you have to wait to buy the stereo?

86. You have just won the lottery. You and your heirs will receive \$25,000 per year forever. What is the present value of this lottery given an 8% discount rate?

87. Homer and Marge wish to buy a new home. The price is \$150,000. Springfield Savings and Loan will lend them the money at a 10% APR for 30 years, with monthly payments to begin in one month. How much will their monthly payments be?

88. Refer to the problem above where Homer and Marge want to buy a new home. The price is \$150,000. Springfield Savings and Loan will lend them the money at a 10% APR for 30 years, with monthly payments to begin in one month. Suppose Homer wants to pay off the loan in 15 years. How much extra must they pay each month to do so?

89. In 1958 the average tuition at an Ivy League school was \$1,800. Thirty years later, in 1988, the average cost was \$13,700. What was the growth rate in tuition?

90. Tom Smith just graduated from college and is thinking about his retirement. He wants to retire at age 50—27 years from now. He assumes that he will spend 35 years in retirements and he has estimated that he will require \$200,000 in each year of his retirement. How much must he deposit each year in an account that earns 12% per year? Assume end of year deposits and withdrawals.

91. Today is Janet's 23rd birthday. Starting today, Janet plans to begin saving for her retirement. Her plan is to contribute \$1,000 to a brokerage account each year on her birthday. Her first contribution will take place today. Her 42nd and final contribution will take place on her 64th birthday. Her aunt has decided to help Janet with her savings, which is why she gave Janet \$10,000 today as a birthday present to help get her account started. Assume that the account has an expected annual return of 10 percent. How much will Janet expect to have in her account on her 65th birthday?

92. Assume that you will receive \$2,000 a year in years 1 through 5, \$3,000 a year in years 6 through 8, and \$4,000 in year 9, with all cash flows to be received at the end of the year. If you require a 14% rate of return, what is the present value of these cash flows?