6. Suppose Ticino could pay \$5 billion per year. How long would it take to pay off the debt, assuming interest continued to be charged at 8.54% APY?
7. Suppose Ticino could pay \$12 billion per year. How long would it take to pay off the debt, assuming interest continued to be charged at 8.54% APY?

Postscript: To the burghers' relief, the judgment was thrown out on appeal. This no doubt restored the confidence of the good burghers of Ticino in the American system of justice!

APPENDIX: THE THEORETICAL BASIS FOR THE NPV RULE

The NPV rule is based on how capital markets help allocate resources efficiently. The crucial variable is the "interest rate," which is often referred to as the opportunity cost of capital. The *opportunity cost of capital* is the price to "rent" money. It is the return the users of funds (for example, borrowers) must pay suppliers of funds (for example, lenders) for the use of their capital. The opportunity cost of capital is important, because it determines who will lend, who will borrow, and the amount of total capital supplied and used.

In Chapter 14, we will formally describe an environment called a *perfect capital market environment*. For now, think of a perfect capital market as a streamlined capital market that glosses over the complexities of real capital markets. In a perfect capital market environment, all prices are fair, so all financial assets have a zero NPV. The expected return of every financial asset equals its required return. Our analysis assumes that there is a single capital market where all users and suppliers of capital can make transactions.

We develop the concepts here as though there is a single opportunity cost of capital we call *the* interest rate. This unique rate is determined through competition among suppliers (lenders) and users (borrowers) of funds. It is the key to how the capital market allocates resources efficiently.

In practice, it seems as though there are many different rates. This is because each market return depends on the risk associated with it and on how long the money will be used. From the Principle of Risk-Return Trade-Off, we know that the higher the risk, the higher the required return. But such differences in risk and return do not affect value. That is the nature of the trade-off. Otherwise, if there were differences in value, there wouldn't be a trade-off—everyone would take the choice with the highest value and not the others. To avoid the complication created by risk-return trade-offs, our analysis assumes that all assets are riskless.

Concerning how long the money will be used, recall from our discussion of the term structure of interest rates in Chapter 3 that generally the longer the commitment, the higher the required return. Like differences in risk, differences in *maturity* lead to alternatives (trade-offs) that are equal in value. Therefore, our final simplifying assumption is that all assets last one year.

The Price of Impatience and the Value of Waiting

If the interest rate is 10% per year and you lend \$10,000 today, you'll get \$11,000 (your principal plus \$1000 interest) one year from today. But if you lend the \$10,000 today, you can't spend it today. You give up that opportunity. In return for waiting a year, however, you'll be able to spend \$11,000, thereby getting an extra \$1000 to spend. In this sense, the 10% interest rate measures the *opportunity cost*. It is the price of impatience and the value of waiting.

Figure 4-A1 shows how the capital market allows people to trade off spending (consumption) now against spending (consumption) in the future. Suppose you have income of C today and C a year from today. Without a capital market, you could spend no more than C today, and no more than C a year from today, assuming there was no way to store any unused

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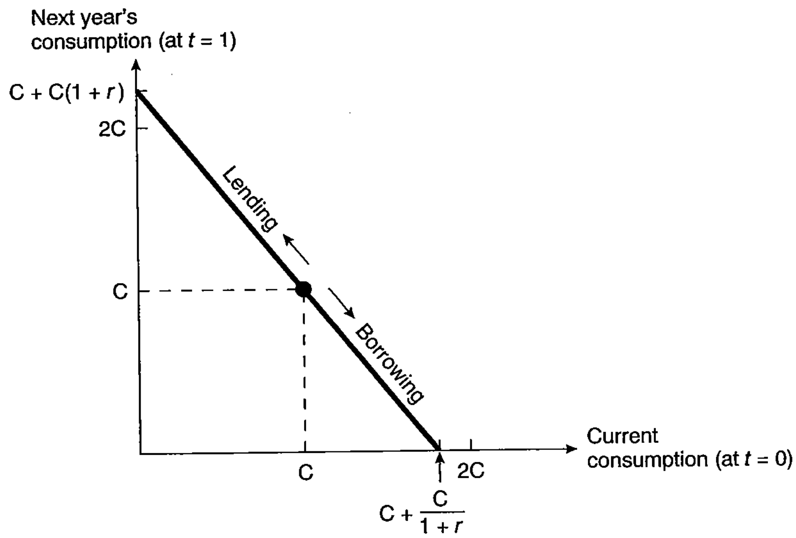


FIGURE 4-A1
All possible consumption (spending) combinations now and next year.

cash. Being so limited could be very inconvenient, especially if you didn't have a regular cash flow each year.

The capital market expands your choices between spending today and spending in the future. The line in Figure 4-A1 shows all the possible combinations of spending today (measured along the horizontal axis) and spending a year from today (measured along the vertical axis). The slope of the line is the rate at which you can trade off current consumption for future consumption, and vice versa. Mathematically, the slope equals $-(1 + r)$, where r is the interest rate. Given your income, you could spend C today and C next year (at $t = 1$). Alternatively, you could lend your entire current income C, spend nothing today, and spend C + C$(1 + r)$ next year. At the opposite extreme, you could borrow the present value of your future income, C/$(1 + r)$, spend C + C/$(1 + r)$ today, and spend nothing next year. Of course, combinations along the line between these two extremes are also possible.

Here is an example to help you see the point. Suppose C = \$10,000 and r = 10%. You can spend \$10,000 today and \$10,000 next year if you don't borrow or lend. Alternatively, you can lend all of this year's income and get to spend \$21,000 (10,000 + 11,000) next year. At the other extreme, you can borrow \$9090.91 (= 10,000/1.1), spend \$19,090.91 now, and have nothing to spend next year. By borrowing or lending, you can take any position along the line in Figure 4-A1.[8]

Smoothing Consumption Patterns

People don't all have the same spending needs and preferences. The capital markets allow people to spend in whatever way they choose, within the limits of their income. In terms of Figure 4-A1, you can be anywhere along the line, but not above it, because you have only \$10,000 per year of income.[9]

Consider two people with the same income C today and C next year (at $t = 1$). Call them Miser and Spendthrift. Miser would like to spend 0.5C today and C + 0.5C$(1 + r)$ next year.

[8] Note that the slope of the line is $-(1 + r) = -1.1$, which equals $-21{,}000/19{,}090.91$.
[9] If you've had some economic theory, you may recognize the line in Figure 4-A1 as an individual's *consumption-possibilities curve.* We could superimpose an *indifference curve* to find the consumption point where the individual would be best off. This point is where the consumption-possibilities curve is tangent to the indifference curve. This is where the marginal rate of time preference (trading off spending now for spending in the future) equals the interest rate.

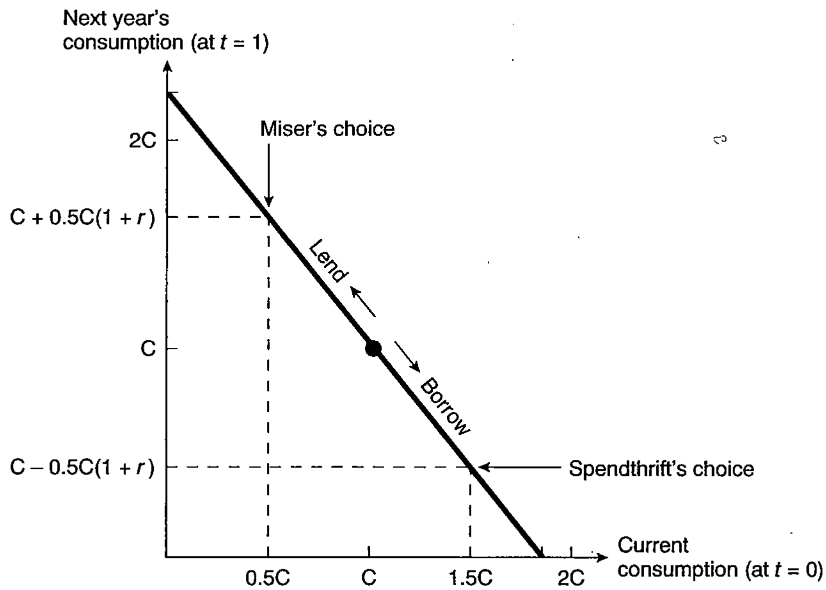


FIGURE 4-A2 Miser's and Spendthrift's consumption (spending) choices.

How can she do this? She lends 0.5C and gets back 0.5C(1 + r) next year. Spendthrift, on the other hand, would like to spend 1.5C today and C − 0.5C(1 + r) next year. Thus he borrows 0.5C today but must pay back 0.5C(1 + r) next year. Figure 4-A2 illustrates Miser's and Spendthrift's consumption (spending) choices.

The capital market lets all individuals have their own preferred spending pattern, within the limit of their total wealth.

All Possible Investment Opportunities

Another way of viewing spending choices is to express them as capital market investment opportunities. By default, this year's spending is, in essence, an investment decision. The less you spend now, the more you invest, and the more you will have next year.

Figure 4-A3 shows the current decision as an investment decision. The line proceeding up and to the right from the origin is a line of all possible investment amounts. If you invest all your current income, C, then next year you will have the investment's future value, C(1 + r). Likewise, an investment of 0.5C will return 0.5C(1 + r), and so on. Thus this capital market investment-opportunities line has a slope of (1 + r).

In addition to capital market investment opportunities, market participants can invest in real assets. Figure 4-A4 presents a real asset investment-opportunities curve, with the opportunities ranked by their expected return from largest to smallest. It shows the combinations of investment today and return next year to be had from investing in real assets.[10] Note that the real asset investment-opportunities curve in Figure 4-A4 is not a straight line. This is because of diminishing returns from investing in real assets. Recall the Principle of Valuable Ideas. Quite simply, some ideas create more value than others. In the extreme, some ideas are actually costly; they will not return enough to be worthwhile.

The opportunity to invest in real assets greatly expands the set of investment possibilities beyond those shown in Figure 4-A3. More significantly, comparing capital market and

[10] It might occur to you that the smoothness of the real asset investment-opportunities curve assumes that investment opportunities can be broken into "very small" pieces. That is, the curve isn't "lumpy" the way it would be if opportunities were large and indivisible, as they typically are.

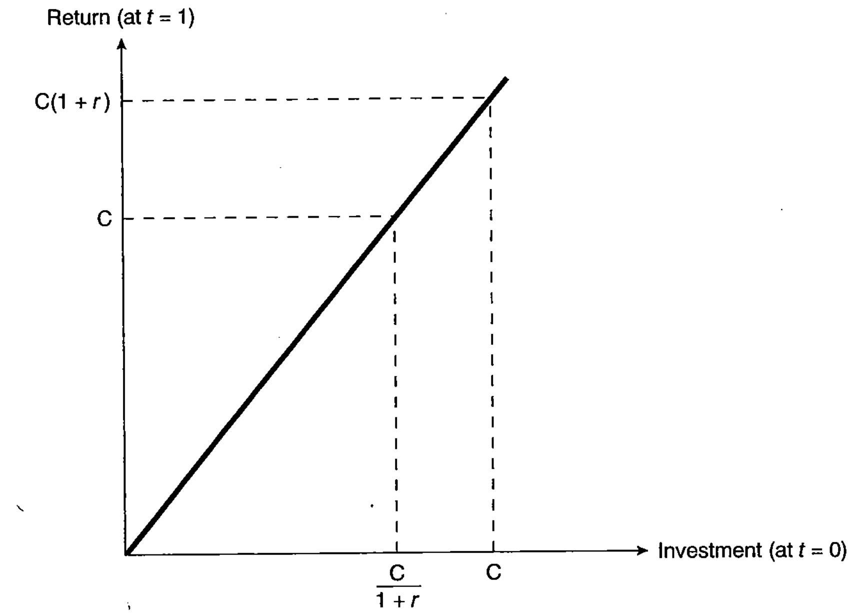


FIGURE 4-A3
The capital market investment-opportunities line.

real asset investment opportunities simultaneously leads us to a very important result: a rule for choosing investments in real assets.

Because of self-interested behavior, people will invest in the most profitable real asset investment opportunity first, in the next most profitable opportunity second, and so on. In Figure 4-A4, the scale is represented in millions of dollars, so the first \$1 million of investment produces a return of \$2.5 million. Investing the second \$1 million produces a return of \$2.25 million. Investing additional \$1 million amounts would produce successively smaller returns. But how much should the *total* amount of investment be?

If you can earn r per period by investing in the capital market, why would you ever invest in a real asset investment opportunity of identical risk that returned less than r? The an-

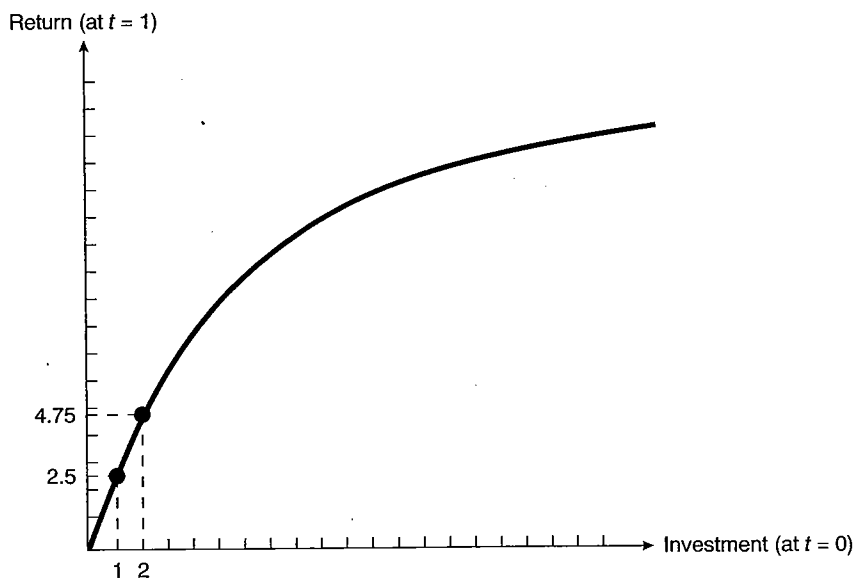


FIGURE 4-A4
The real asset investment-opportunities curve (in millions of dollars).

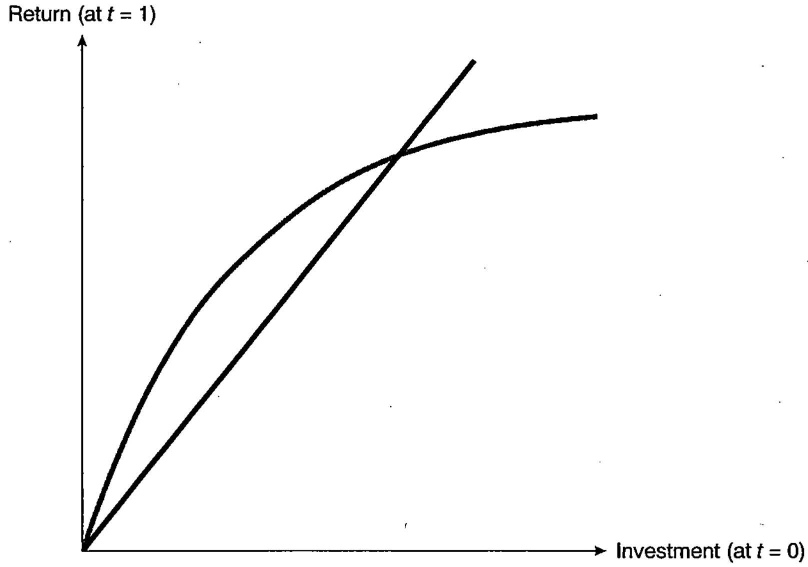
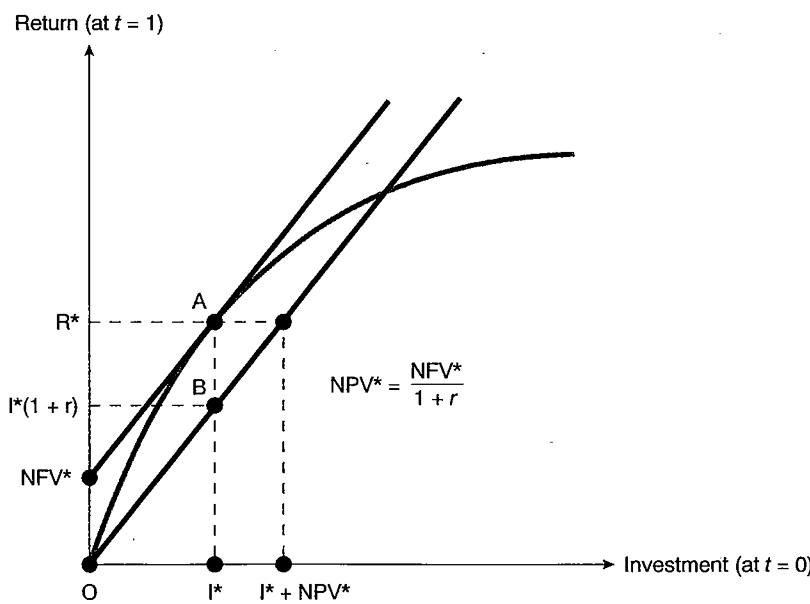


FIGURE 4-A5 Capital market investment-opportunities line superimposed on the real asset investment-opportunities curve.

swer is that you wouldn't. And this is exactly where the net present value (NPV) rule originates. *Don't buy an asset that isn't worth what you have to pay for it.* In other words, don't buy a negative-NPV asset.

Figure 4-A5 superimposes the real asset investment-opportunities curve on the capital market investment-opportunities line. If people use the NPV rule just stated, they will not invest in negative-NPV real assets—those that do not return at least r. These opportunities are where the slope is less than $(1 + r)$. But they should invest in all the opportunities up to that point—those that return at least $(1 + r)$. Therefore, the optimal total investment is found by putting a line parallel to the capital market investment-opportunities line, tangent to the real asset investment-opportunities curve. Figure 4-A6 illustrates this concept by adding this parallel line.

FIGURE 4-A6 The optimal total investment, I*, and the corresponding increase in present value, NPV*.



In Figure 4-A6, the optimal total investment is I*, and the total return at $t = 1$ is R*. A fair total return (expected equals required) would be $I^*(1 + r) = R^* - NFV^*$ (because the triangles AR* NFV* and BOI* are congruent). Consequently, the total excess return at $t = 1$ (the net future value) will be NFV*. NFV* has a present value of NPV* (= $NFV^*/[1 + r]$).

Our description works if you have *enough* money to invest in a positive-NPV real asset. But suppose you have some money to invest, but not that much? Does this mean you have to give up all real asset investment opportunities? No! The capital market allows both positive and negative investment—that is, lending *and* borrowing. Therefore, when you have found a positive-NPV real asset but lack the cash to make the investment, you can borrow the money in the capital market.[11]

Returning to Miser and Spendthrift, we can see how investing in real assets makes them *both* better off. The value added by investment I* is NPV* in terms of $t = 0$ dollars and is NFV* in terms of $t = 1$ dollars. We can express an individual's value from investing in real assets as IPV at $t = 0$ and as IFV at $t = 1$. This extra value adds to each individual's income. Figure 4-A7 is adapted from Figure 4-A2 by including the value added from investments in real assets. In Figure 4-A7, we can see how investing in real assets can make both Miser and Spendthrift better off. By investing in positive-NPV real assets, they will be able to spend more—both now *and* next year.

It is vital to understand that Spendthrift will borrow (get some type of financing for) the extra money he needs to invest—on top of the amount he was going to borrow anyway to spend (consume) now. But despite this, Spendthrift will be better off because he will get the NPV from those investments (IPV in Figure 4-A7). Miser is also better off, because some of

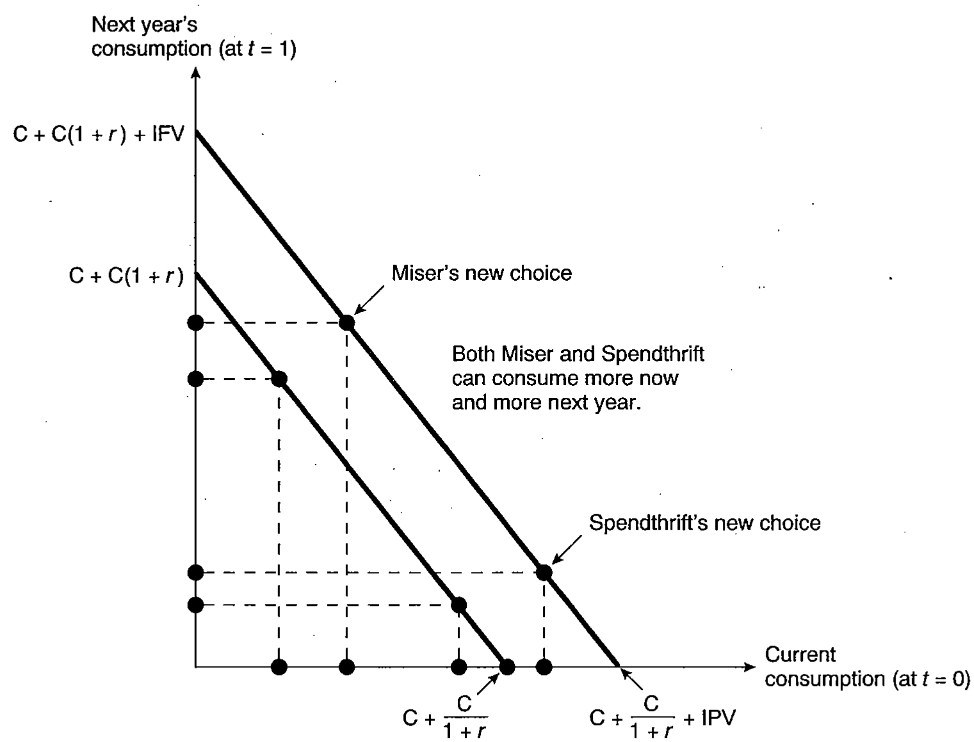


FIGURE 4-A7
The individual's gain from investing in real assets.

[11] We are using the term *borrowing* to mean any use of capital with any type of financing. The risk-return trade-off makes the various types equivalent in value.

the money she was going to save (lend) will be invested in positive-NPV real assets. Therefore, a key point of the analysis is that the NPV rule applies to all individuals, regardless of their personal spending choices.

The capital markets bring together the combined choices for investing in real and financial assets and borrowing for either spending (consumption) or investment purposes. Thus the capital markets play a crucial role, because they determine the trade-offs among all these choices. These trade-offs are embodied in, or represented by, the rate of interest. The rate of interest, then, is a measure of the opportunity cost of choosing one alternative over another. The rate of interest is the opportunity cost of capital.

Fisher Separation

The NPV rule, which was derived by the American economist Irving Fisher two-thirds of a century ago, means the optimal amount of investment in real assets does not have anything to do with individual spending preferences. As long as people have free access to the capital markets (that is, that they can supply or use capital freely at the market interest rate), each person can choose his or her own spending (consumption) pattern.

This means that a firm's investment decisions are *separate* from individual owners' spending preferences. This result, that investment choices are separate from individual spending preferences, is called **Fisher's separation theorem**.

Fisher's separation theorem has an important implication for firms, some of which have literally hundreds of thousands of shareholders. Such firms are managed by professional managers. The ownership and control of these firms are separated, as we discussed in our set-of-contracts model. Fisher's separation theorem provides a unifying goal for the firm, a goal that the firm's shareholders can all agree on. All the owners, regardless of their spending preferences, will be best off if the firm invests in positive-NPV assets, because all will get their share of the NPV, so each will ultimately be able to spend more. Therefore, *a firm should maximize the NPV of its investments*. This, then, is the basis for the NPV rule.

Fisher's separation theorem—and therefore the NPV rule—rests on the assumption of a perfect capital market environment. To the extent that real capital markets deviate from this, Fisher's separation theorem may only be a first approximation. In Chapter 14, we discuss the existence of capital market imperfections and their implications for the firm.

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