Diversification and Risky Asset Allocation

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Diversification

• Intuitively, we all know that if you hold many investments
  • Through time, some will increase in value
  • Through time, some will decrease in value
  • It is unlikely that their values will all change in the same way
• Diversification has a profound effect on portfolio return and portfolio risk.
• But, exactly how does diversification work?

Diversification and Asset Allocation

• Our goal in this chapter is to examine the role of diversification and asset allocation in investing.
• In the early 1950s, professor Harry Markowitz was the first to examine the role and impact of diversification.
• Based on his work, we will see how diversification works, and we can be sure that we have "efficiently diversified portfolios."
  – An efficiently diversified portfolio is one that has the highest expected return, given its risk.
  – You must be aware that diversification concerns expected returns.

Expected Returns, I.

• Expected return is the "weighted average" return on a risky asset, from today to some future date. The formula is:
  \[ \text{expected return} = \sum p_i \times \text{return}_i \]
• To calculate an expected return, you must first:
  – Decide on the number of possible economic scenarios that might occur.
  – Estimate how well the security will perform in each scenario, and
  – Assign a probability to each scenario
  – (BTW, finance professors call those economic scenarios, "states.")

Expected Risk Premium

• Recall:
  \[ \text{Expected Risk Premium} = \text{Expected Return} - \text{Riskfree Rate} \]
• Suppose riskfree investments have an 8% return. If so,
  – The expected risk premium on Jpod is 12%
  – The expected risk premium on Starcents is 17%
• This expected risk premium is simply the difference between
  – The expected return on the risky asset in question and
  – The certain return on a risk-free investment

Calculating the Variance of Expected Returns

• The variance of expected returns is calculated using this formula:
  \[ \text{Variance} = \sigma^2 = \sum p_i \times (\text{return}_i - \text{expected return})^2 \]
• This formula is not as difficult as it appears.
• This formula says to add up the squared deviations of each return from its expected return after it has been multiplied by the probability of observing a particular economic state (denoted by \( \sigma \)).
• The standard deviation is simply the square root of the variance.
  \[ \text{Standard Deviation} = \sigma = \sqrt{\text{Variance}} \]
Portfolios

- Portfolios are groups of assets, such as stocks and bonds, that are held by an investor.
- One convenient way to describe a portfolio is by listing the proportion of the total value of the portfolio that is invested into each asset.
- These proportions are called portfolio weights.
  - Portfolio weights are sometimes expressed in percentages.
  - However, in calculations, make sure you use proportions (i.e., decimals).

Portfolios: Expected Returns

- The expected return on a portfolio is a linear combination, or weighted average, of the expected returns on the assets in that portfolio.

The formula, for "n" assets, is:

\[ E(R_p) = \sum_{i=1}^{n} w_i \times E(R_i) \]

In the formula:
- \( E(R_p) \) = expected portfolio return
- \( w_i \) = portfolio weight in portfolio asset \( i \)
- \( E(R_i) \) = expected return for portfolio asset \( i \)

Variance of Portfolio Expected Returns

- Note: Unlike returns, portfolio variance is generally not a simple weighted average of the variances of the assets in the portfolio.
- If there are "n" states, the formula is:

\[ \text{VAR}(R_p) = \sum_{s=1}^{n} \left[ P_s \times \left( E(R_{p,s}) - E(R_p) \right)^2 \right] \]

- In the formula, \( \text{VAR}(R_p) \) = variance of portfolio expected return
  - \( P_s \) = probability of state of economy, \( s \)
  - \( E(R_{p,s}) \) = expected portfolio return in state \( s \)
  - \( E(R_p) \) = portfolio expected return

- Note that the formula is like the formula for the variance of the expected return of a single asset.

Diversification and Risk, I.

Why Diversification Works, I.

- Correlation: The tendency of the returns on two assets to move together. Imperfect correlation is the key reason why diversification reduces portfolio risk as measured by the portfolio standard deviation.
  - Positively correlated assets tend to move up and down together.
  - Negatively correlated assets tend to move in opposite directions.
- Imperfect correlation, positive or negative, is why diversification reduces portfolio risk.
Why Diversification Works, II.

- The correlation coefficient is denoted by $\text{Corr}(R_A, R_B)$ or simply, $\rho_{A,B}$.
- The correlation coefficient measures correlation and ranges from:
  
  From: -1 (perfect negative correlation)
  Through: 0 (uncorrelated)
  To: +1 (perfect positive correlation)

Why Diversification Works, III.

Calculating Portfolio Risk

- For a portfolio of two assets, A and B, the variance of the return on the portfolio is:

$$\sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_Ax_B \text{COV}(A,B)$$

Where: $x_A = \text{portfolio weight of asset A}$
$x_B = \text{portfolio weight of asset B}$
such that $x_A + x_B = 1$.

(Important: Recall Correlation Definition!)

The Importance of Asset Allocation, Part 1.

- Suppose that as a very conservative, risk-averse investor, you decide to invest all of your money in a bond mutual fund. Very conservative, indeed?

  Uh, is this decision a wise one?
The various combinations of risk and return available all fall on a smooth curve.

This curve is called an *investment opportunity set*, because it shows the possible combinations of risk and return available from portfolios of these two assets.

A portfolio that offers the highest return for its level of risk is said to be an *efficient portfolio*.

The undesirable portfolios are said to be *dominated* or *inefficient*.

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**The Markowitz Efficient Frontier**

- The Markowitz Efficient frontier is the set of portfolios with the maximum return for a given risk AND the minimum risk given a return.

- For the plot, the upper left-hand boundary is the Markowitz efficient frontier.

- All the other possible combinations are inefficient. That is, investors would not hold these portfolios because they could get either
  - more return for a given level of risk, or
  - less risk for a given level of return.