At-the-Money Stock Options, Incentives, and Shareholder Wealth

Don M. Chance and Tung-Hsiao Yang*

Current version: February 9, 2008

JEL Classification codes: G32, G39, M52

Key words: executive stock options, stock options, executive compensation, incentives

William H. Wright, Jr. Endowed Chair for Financial Services, Louisiana State University and Assistant Professor of Finance, National Chung Hsing University, Taiwan. Contact author: Chance, Department of Finance, 2163 CEBA, Louisiana State University, Baton Rouge, LA 70803; 225-578-0372, 225-578-6366 (fax), dchance@lsu.edu. This paper was presented at the Eastern Finance Association and the Northern Finance Association. The authors appreciate the helpful comments of Fritz Koger, Sam Tian, Rainer Andergassen, Anna Dodonova, Cliff Stephens, and seminar participants at the Korea Advanced Institute for Science and Technology, West Virginia University, Louisiana State University, and York University.
Properly designed incentives should reward management if and only if it succeeds in increasing shareholder wealth. We show that conventional at-the-money stock options do not achieve this objective, because they allow management to benefit from value created before the options were awarded. Consequently the cost of the options can exceed any value created for shareholders, and management can even benefit from negative-NPV projects. We also find that with at-the-money options, dividends are undesirable to management unless the strike is adjusted while repurchases are quite favorable to management. These problems are solved by indexing the exercise price to the cost of capital, a type of option that has been discussed lightly in the practitioner literature. Empirical estimates reveal that the difference in value between at-the-money options and options indexed to the cost of capital is about one-quarter percent of equity value but about 42 percent of option value and about 18 percent of total compensation. This figure is positively and significantly related to measures of free cash flow and overinvestment, as predicted by the model. If options are viewed strictly as incentives, this figure suggests a substantial waste of shareholder value, but if options are viewed as incentives and compensation, it provides a reasonable estimate of the latter and by implication, the former.
At-the-Money Stock Options,
Incentives, and Shareholder Wealth

The widespread use of executive stock options has created much controversy in the corporate world. Barely a month passes without at least one media report on the millions of dollars earned when an executive exercises stock options. Controversial practices, such as backdating and repricing have brought considerable media attention and negative publicity to many firms. The first quarter of 2006 saw American firms for the first time recording stock options as an expense, leading to continuing controversy over whether volatility estimates are systematically downward biased to boost earnings.¹

In this paper we propose that in addition to these controversies and practices, stock options have a subtle and unrecognized problem with respect to their role as an incentive device. For example, a good incentive should reward management if and only if shareholder wealth increases as a result of actions taken by management after the incentive is granted. We show that traditional stock options fail this test. Specifically, the cost of these instruments can exceed value created when management adds a positive net-present-value (NPV) project to the firm’s portfolio of assets. Moreover, we show that management can even benefit from adding a negative-NPV project, though we will show that there is a better alternative for management. We also show why management will resist the payment of dividends unless the exercise price is adjusted by the dividend and why management would prefer share repurchase over dividends as a use of the available cash. Finally, we demonstrate that there is an alternative structure that solves these problems. An interesting by-product of our analysis is that we can decompose the value of an at-the-money option into salary, which compensates for past performance, and incentives, which are based on future performance.

Our model is extremely simple and integrative in that it captures the essential characteristics of a firm that offers options, while incorporating the cost of the options into the value of the firm. We require only one significant constraint, which we show is met in a simple principal-agent framework. Our model directly reflects the benefit to shareholders and management and illustrates the conflicts created by using options. While we first develop the model in a world of deterministic cash flows, we later show how uncertainty creates new opportunities for management to create value and how the solution under certainty is supported under uncertainty.

¹See Jones (2006).
Section I presents an overview of the problem and the literature. Section II sets up the model, and Section III presents the main results for traditional options. Section IV shows how a re-design of the option solves the problem. Section V presents empirical estimates of the cost of this problem, and Section VI provides conclusions.

I. The Problem with Traditional Stock Options

Following the classic Jensen-Meckling (1976) theory of agency costs, conflicts between shareholders and management have become frequent issues in the finance literature. Because management is a consumer, an investor, and an expected utility maximizer, it is reasonable to expect that management will make executive decisions in such a manner that its own expected utility is maximized. As a result, managerial risk aversion is likely to influence decisions about corporate investment, and possibly financing, thereby leading to a more diversified asset portfolio and less risk at the corporate level, regardless of shareholder desires.

DeFusco, Johnson, and Zorn (1990) were among the first to argue that stock options could be used to motivate managers to take greater risks. Others have taken somewhat contrary views. Carpenter (2000), for example, shows that a manager’s appetite for risk can be either greater or less as a result of options. Managers can even make the firm have less risk than the managers would take in their own personal portfolios. Ross (2004) demonstrates that whether options increase the firm’s risk depends on whether options shift the domain of the manager’s utility function to a more or less risk-averse section. Nohel and Todd (2004) find that pay-for-performance sensitivity and firm value are not monotonically related in the presence of options. They show that it is possible that even small option awards can cause management to take excessive risks.

Brisley (2006) finds that deep in-the-money options reduce the incentives and lead managers to reduce risk. He also shows that vesting should be done in a progressive manner proportional to the stock price, the consequences of which are that more options are retained when the incentives are greatest and more exercised when the incentives are smallest. Nonetheless, in the U. S., at-the-money options have been the preferred instrument. Hall and Murphy (2000) show that at-the-money options offer the best trade-off between cost and incentives. In contrast, however, Palmon et al (2004) argue that out-of-the-money options are better if managerial effort is considered.

Anecdotal evidence suggests that accounting treatment is another reason why options, and especially at-the-money options, have been so widely used. Until recently, the Financial Accounting Standards Board had permitted firms to avoid expensing
options under old APB 25 of 1972, which essentially expensed options at their exercise values as of the grant date. With nearly all options granted at-the-money, the expense was de facto zero. Under FAS 126R, effective the first quarter of 2006, firms are required to record an expense at grant date based on an estimate of the option’s value. Naturally much controversy has arisen over this new rule. Firms that rely heavily on options expressed much opposition before the rule was passed and have continued to criticize the practice.

As noted in the introduction, options have also led to controversy over such practices as backdating (Narayanan and Seyhun (2007)), repricing (e.g., Coles, Daniel, and Naveen (2005)), reloading features (e.g., Dybvig and Loewenstein (2003)), timing grants (Yermack (1997)), and rescissions (Brenner, Sundaram, and Yermack (2005)). In spite of all of the controversies, corporate America, in particular the technology industry, has clearly embraced the use of options. By enabling management to benefit from share price increases, options on the surface help align managerial and shareholder interests. If options are also used as compensation, they preserve cash, a probably reason why many firms with shortages of cash rely heavily on them.

In this paper, we show that at-the-money options have a subtle problem that clarifies move vividly why they seem to be so popular with management. We find that they allow management to capitalize on successes achieved before the options were awarded. Hence, they partially reward management for past successes.

II. Model Structure

Our model is initially derived in a world of limited certainty, by which we mean that the cash flows of a firm are assumed to be known but all information about the future is not known. This framework is sufficient to illustrate many of the most important points. As Damodaran (2005) notes, “if two projects have the same net present value, firms should be indifferent between them (p. 44).” Thus, we should not care whether a project’s NPV is derived from risky cash flows discounted at a risk-adjusted rate or risk-free cash flows discounted at the risk-free rate. We incorporate uncertainty, however, in the form of allowing only for the possibility but not certainty that management will discover new projects that increase shareholder wealth. Cash flow uncertainty will introduce some complexities that are addressed in a later section. It should be noted that these complexities reveal some additional insights into the proper role of incentive contracts for management. But for now, uncertainty will strictly derive from the question of whether management is successful in finding positive net-present-value projects.
A. Basic Setup

Consider an all-equity firm in a one-period world, starting at time 0 and ending at time 1. The firm has existing assets consisting of cash in the amount of $C_0$ and other assets invested in various projects, producing certain cash of $C_1$ at time 1. These risk-free cash flows are discounted at the risk-free rate, $r - 1$, which obviously is the company’s cost of capital.

The objective of the firm is naturally to increase shareholder wealth, which occurs if management can add projects with positive net present value. To induce managerial effort, the board offers options to management. In this model, options are granted and management then has an instant to find a positive-NPV project. We will also consider alternative uses for the cash, such as dividends and share repurchase, which can be done if desirable projects are not found. In addition, we will look at the use of outside financing to raise funds for investment. We assume that the investment opportunity set is fixed, but the board has no knowledge of the opportunities.

Initially, let us disregard the use of options and illustrate the model in a world in which management need not be incentivized to find a positive-NPV project. As noted, the firm’s existing projects generate cash of $C_1$ at time 1 and, therefore, have a present value of $C_1 r^{-1}$, and, therefore, the company has cash on hand of $C_0$. The value of the firm before the options are granted is

$$S_0 = C_0 + C_1 r^{-1}.$$ (1)

If management finds a positive-NPV project, $C_0$ is invested and creates cash flow at time 1 of $\Delta C_1$. Of course, a project with positive net present value means that $\Delta C_1 r^{-1} > C_0$.

The firm’s total cash flow at time 1 will then be $C_1 + \Delta C_1$. All of this cash is distributed to shareholders and the company terminates. The new stock price at time 0 will be

$$S_0' = C_1 r^{-1} + \Delta C_1 r^{-1}.$$ (2)

Of course, $S_0'$ exceeds $S_0$ as a result of the positive value added of $\Delta C_1 r^{-1} - C_0$. Naturally these results are just standard paradigms from the theory of finance.

B. Options Awarded to Incentivize

To increase the likelihood that the manager will add value, let us assume that the Board of Directors grants the manager $\gamma$ options where $\gamma$ is the number of options divided by the number of outstanding shares. All results require only the restriction

---

2Extension of the model to a multiperiod setting can be done by letting the time 1 cash flow also include the value of all remaining cash flows. We discuss some aspects of multiperiod analysis later.

3As with $C_1$, $\Delta C_1$ can also reflect the value of future cash flows for a multiperiod project.
that the Board awards fewer options than there are shares. Thus, $\gamma < 1$. Later we will show that this constraint is upheld in a simple principal-agent model.

To avoid some complexity that would be created by the issuance of new shares upon exercise, let us treat the options as cash-settled, which are typically called share appreciation rights. Because the options are settled in cash and because we liquidate the firm after all value has been distributed to shareholders, we must carefully define how the payoff of the options is determined.\footnote{Because the firm is liquidated at time 1, options that settle in shares would have the same economic impact as cash-settled options. But because the stock price would be zero upon liquidation, we must carefully define the rule under which the option payoff is made. Also, note that if $C_i$ and $\Delta C_i$ reflect the value at time 1 of cash flows beyond time 1, then we assume that these projects could be liquidated.}

We assume that the option payoff will be deducted from the available cash at time 1. Hence, the payoff per option is $\text{Max}(0, C_1 + \Delta C_1 - X)$ where $X$ is the exercise price. After paying off the options, the remaining cash will be paid out to the shareholders and the firm liquidated. At this point, it should be apparent that this model is integrative, unlike many other models of stock options usage in a firm. In many models, the cost of options is not directly incorporated into the value of the firm. Moreover, the option value depends on corporate cash flows and shareholder wealth depends on corporate cash flows and option value. This circularity must be built into the model. Also, note that we can refer to the value and cost of the option as the same. The value to management is the same as the cost to the firm. Differences arise only due to such factors as illiquidity and vesting, which are not considered here.\footnote{These factors are relevant but their importance arises for reasons unrelated to the issues addressed in this paper. In addition, the one-period nature of the setup precludes any concern over liquidity or vesting issues.}

Now we must establish a criterion that identifies whether the option grant was successful for the shareholders. It seems natural to assume that if management is granted options for the purpose of adding value for the shareholders, then those options should expire with value if management creates value for the shareholders and should expire worthless if management is unable to find a value-increasing project. If management is successful, it should share in the new value created and if not, it should not benefit. With certainty of cash flows at time 1, these characteristics are met if the option value at time 0 is positive if management is successful and zero if it is not. Define $v_0$ to be the value of the option today, which represents the discounted expected value of the option payoff at expiration.\footnote{We need not make any assumptions about how the option is valued. Therefore, the question of whether the Black-Scholes model is appropriate is not important in this context.} With cash flow certainty, we see that

\begin{equation}
\begin{aligned}
v_0 &= r^{-1} \begin{cases} 
C_1 + \Delta C_1 - X & \text{if } C_1 + \Delta C_1 > X \\
0 & \text{otherwise}
\end{cases} 
\end{aligned}
\end{equation}
In other words, the terminal value of the option is known as soon as the incremental cash flow $\Delta C_t$ is known. With cash flow certainty, the initial value of the option is then known. As noted, a desirable property is that $v_0$ be positive if management increases shareholder wealth and zero if not.

Let us also note that we assume that management’s reservation price is covered in the net cash flow, $C_1$. The options are intended to represent pure incentives. Because management does not pay for the options, it benefits whenever the options have positive value.

We also assume that management is unable to divert resources from one project to another. In other words, it can do nothing to increase the incremental cash flow at the expense of the existing cash flow. This assumption is guaranteed by the certainty associated with the existing cash flow at time 1 but is worth remembering. We also assume that the market does not react to the awarding of the options.

III. Model Results

We start by examining what we call traditional options, in which the exercise price is the current stock price, $X = S_0$. The results are stated in the form of propositions.

A. The Firm Makes Capital Investments

In this section we assume that management causes the firm to invest its cash into capital projects. Let us first assume that management finds a positive net-present-value project.

PROPOSITION 1: If management finds a positive-NPV project, the option will have value.

Proof: See Appendix.

Thus, management will benefit if it finds a positive-NPV project. Clearly such a result is a necessary condition for the options to serve as an incentive.

Now let us examine the question of whether the cost (value) of the options can exceed the net present value, thereby wiping out any shareholder gains. We measure the cost of the options by their payoff at time 1.

---

7As we show later, part of the value of an at-the-money option is based on cash flows that were in place before the options are awarded. As such, part of the option value can be viewed as compensation, possibly compensation in excess of the manager’s reservation price.

8Of course, management cannot exercise or sell the options so it cannot realize the value until later.
PROPOSITION 2: The cost of the options awarded to management can exceed the value of a positive-NPV project that management finds and the firm undertakes.

Proof: See Appendix.

As the proof indicates, whether the cost exceeds the NPV depends on the number of options awarded. But this maximum number of options is a function of the NPV of the new project. Whether management can find a new project with positive NPV and the NPV of that new project are the uncertain state variables. Because the board of directors would not know before granting the options whether management would be successful and if so, how much value would be created, it would not be possible for the board to be sure that it awards a sufficiently small number of options so that cost does not exceed NPV.

Thus, even if management is successful in finding a project with positive NPV, the shareholders can end up paying management more than the NPV. Hence, if options are awarded to incentivize management, the traditional NPV rule is not an optimal decision rule for the shareholders.

We now examine whether the options can have value even if the project has a negative net present value.

PROPOSITION 3: The option can have value even if management selects a negative-NPV project.

Proof: See Appendix.

This result implies that if management cannot find a positive-NPV project, it can still benefit from a negative-NPV project. Thus, in the absence of monitoring, management can clearly benefit from actions that are highly detrimental to the shareholders. It is even possible to show that the options can have value when the project has a negative internal rate of return. Management might choose a negative-NPV for the purpose of empire-building. That said, however, we are not implying that management would automatically select a negative-NPV project. As we will show, there are several other choices for management that are better, though they would not capture whatever benefits management might perceive as associated with empire-building.

9 The option has value if $C_i + \Delta C_i > S_0$. Define the IRR to be $i - 1$ where $\Delta C_i = C_i$. Substituting this definition shows that $i$ can be less than one meaning a negative IRR, with positive option value.
We now summarize and illustrate these results in Figure 1. First, define the NPV as $b = \Delta C_1 r^1 - C_0$. Shareholders and management benefit from all positive-NPV projects, that is, where $b > 0$. From management’s standpoint, the positive payout of the options is $C_1 + \Delta C_1 - (C_0 + C_1 r^1) > 0$ or $C_1 > -r C_0 - br^2/R$ where $R = r - 1$. Management and shareholders would find all projects with positive NPV beneficial, but there is a region in which negative NPV projects are beneficial to management. Hence, rather than solving an agency problem, traditional stock options can create an agency problem.

The reason for this overinvestment problem is that the options pay off based not only on the incremental cash flow but also on the cash flow that existed before the options were awarded, $C_1$. Options allow management to benefit not only from any new value created but also from value that existed before the options were awarded. Hence, negative-NPV projects can be supported by value existing before the options were awarded. This result comprises Proposition 4.

**PROPOSITION 4:** If managers are incentivized with traditional options, they will receive a portion of the gains from projects that the firm had in place before the options were awarded.

*Proof:* See Appendix.

This finding suggests that options have a deficiency with respect to their role as an incentive device. Management would prefer the project with the highest NPV, because the option payoff varies directly with $\Delta C_1$, but management could benefit if the project did not have positive NPV. Monitoring could possibly prevent management from accepting negative-NPV projects, but it is widely accepted that monitoring is not completely effective. But as we will see later, there is another alternative to a negative-NPV project that is better for management.

**B. The Effect of Dividends**

If there are no positive-NPV projects, the obvious alternatives are to pay dividends, repurchase shares, or invest in a zero-NPV project, such as purchasing financial assets. Let us now consider these alternative uses of funds, starting with dividends. Thus, no capital investment is made, and the cash, $C_0$, is paid out as a dividend. We state the effect of dividends as Proposition 5.
PROPOSITION 5: If cash is not invested in new projects but is instead paid out as a dividend, the option will have value only if the dividend is smaller than the time value of the cash flow from the existing projects.

The proof is simple. With no capital investment, the option will have value only if $C_1 > S_0$, which equivalent to $C_1 - C_1 r^1 > C_0$. Thus, if a dividend is paid, the option can have value only if the left-hand side exceeds the right-hand side, the latter being the dividend. The left-hand side is the time value of the cash flow from existing projects. If the dividend ($C_0$) is sufficiently small relative to this time value, it is possible for the option to have value. But if not, the option will not have value. In other words, the dividend must be “small” relative to the time value of the cash flow from existing projects. Thus, management will prefer that companies pay low dividends or none at all. Empirical evidence is consistent with this finding: options seem to make management more averse to paying dividends.\textsuperscript{10}

A modification to the option that can reduce this problem is for the exercise price to be adjusted by the dividend. This feature, which is common in over-the-counter options markets, effectively makes the option dividend-protected. In this case, the strike would be set at the ex-dividend stock price of $S_0 - C_0 = C_1 r^1$. This adjustment leads to Proposition 6.

PROPOSITION 6: If cash is not invested in new projects but is instead paid out as a dividend and the strike price of the option is adjusted downward by the amount of the dividend, the option will always have value.

The proof is simple. Now the criterion for the option to have value is just $C_1 > C_1 r^1$, which clearly holds. Thus, the adjustment of the option strike over-corrects for management’s aversion to dividends.\textsuperscript{11} From the shareholders’ perspective, a dividend-adjusted strike is not desirable because management benefits even though it was not successful in identifying a positive-NPV project. But if management is powerful enough to control the dividend decision, it may be in the shareholders’ interests to use an adjusted strike. Although dividend protection allows management to profit from its options when it does not find a positive-NPV project, it does at least remove the

\textsuperscript{10}Brown, Liang, and Weisenmer (2004) find that both before and after the 2003 dividend tax cut, companies whose executives have large option holdings are less likely to increase dividends. Also, Lambert, Lanen, and Larcker (1989) hypothesize that firms that adopt executive stock option plans will reduce dividends below what the dividends otherwise would have been. Their empirical results in a sample of about 200 firms over the 1967-1987 period are consistent with this conjecture. The reduction in dividends relative to expectations is largest for cases in which the executive has the most to gain by a reduction in dividends. In short, dividends tend to be lower the more stock options firms use. Fenn and Liang (2002) report similar results.

\textsuperscript{11}Yet another possibility is to set the strike at a level in which the option expires at-the-money if no positive-NPV project is found and a portion of the funds are invested in cash with the remainder paid out as dividends.
impediment to management support of dividends when positive-NPV projects are unavailable.\textsuperscript{12} In practice, however, it is not common in most countries to adjust the strikes of executive stock options for dividends.\textsuperscript{13}

\textit{C. The Effect of Share Repurchase}

Now let us consider share repurchase, the first obvious alternative to dividends. We now state Proposition 7.

\textbf{PROPOSITION 7: If firms offer at-the-money options and repurchase shares, management will always benefit from the options.}

\textit{Proof:} See Appendix.

Ignoring taxes, it is well-known that dividends and share repurchases are equivalent from the shareholders’ point of view. It is apparent, however, that that in the presence of options, they are not equivalent from management’s point of view. Unless the strike is adjusted for dividends, management will clearly prefer share repurchase, a result consistent with empirical research.\textsuperscript{14}

\textit{C. The Effect of Financial Investment}

An alternative project with zero net present value is for the firm to invest its cash in financial assets with the same risk as the firm’s existing assets or investment in a merger with no synergy. We will treat these and any other type of capital investment as the same and simply examine the financial investment case.

\textbf{PROPOSITION 8: If firms offer at-the-money options and invest in zero net-present-value projects, management will always benefit from the options.}

Recall that the option has value if \((C_1 + \Delta C_1 - (C_0 + C_1 r^1))\) is positive and zero otherwise. With zero \(NPV (\Delta C_1 = C_0 r)\), this term becomes \(C_1 - C_1 r^1 + C_0(r - 1)\), which is clearly positive, so management benefits even though it did not add value for the shareholders.

\textit{D. Summary of Uses}

Table 1 summarizes the benefit to management and the benefit to the shareholders from each use of the cash. Though these expressions are complex, they

\textsuperscript{12}There is yet another alternative to full strike adjustment and no strike adjustment. A partial strike adjustment could be derived that would set the strike at precisely the level that the option would expire at-the-money if no new positive-NPV project is found.

\textsuperscript{13}Adjustment of the strike for dividends is evidently done in Finland. See Pasternack and Rosenberg (2003).

\textsuperscript{14}See Kahle (2002), Weisbenner (2000) and Jolls (1998), who find that share repurchases are more widely used by firms that use stock options and are more widely used the more stock options outstanding.
include certain common terms and can be simplified greatly. For example, each of the terms for the benefit to management includes the expression $\gamma(C_1 - C_1 r^i)$, while each of the terms for the benefit to the shareholders includes the expression, $(1 - \gamma)C_1 + \gamma C_1 r^i$. Removing these common expressions makes it easier to see which method is preferred.

The preference for shareholders and management varies according to the values of $\Delta C_i$, $C_0$, and $r$. There are seven conditions that must be considered. Let us denote the five choices for use of the cash as CI (capital investment), D (dividends, no strike adjustment), DSA (dividends with strike adjustment), SR (share repurchase), and FI (financial investment). Table 2 shows the order of preference of management and shareholders for each use under each of nine collectively exhaustive conditions.

The first and only nexus of agreement between management and the shareholders is Condition A(i), the existence of a positive-NPV project ($C_0 < \Delta C_i r^i$) in which $(1 - \gamma)\Delta C_i > C_0 r$. The term $(1 - \gamma)\Delta C_i$ is the portion of the incremental cash flow retained by the shareholders (i.e., after paying out the option payoff). When what the shareholders keep exceeds the opportunity cost of cash invested, the shareholders clearly benefit, as would management. The two other positive-NPV outcomes, however, are undesirable for the shareholders. In Condition A(ii) the portion of the incremental cash flow retained by the shareholders is less than the opportunity cost of cash invested. In Condition A(iii) the portion of the incremental cash flow retained by the shareholders is less than cost of capital, adjusted by the capital that management contributes in exercising its options. Conditions A(ii) and A(iii) are clearly undesirable to the shareholders and they would prefer dividends, but management and the shareholders differ on the alternatives beyond dividends. For Condition A(ii), the second choice of shareholders would be capital investment over a strike-adjusted dividend, while for Condition A(iii) the second choice of shareholders would be a strike-adjusted dividend over capital investment. Of course, in both cases, management prefers capital investment.

All remaining cases involve zero or negative NPV. In each case, management prefers financial investment, while shareholders prefer dividends. In all such cases, the shareholders would even find strike-adjusted dividends preferable to share repurchase, which is preferable to financial investment. Management’s second choice would be share repurchase in all such cases. The only point of agreement is in Conditions F and G, in which where the incremental cash flow is negative or zero. In that case, shareholders and management rank capital investment last, for the obvious reason that neither benefits from zero or negative incremental cash flows. Note also that even though we found that
management can benefit from a negative-NPV project, it should prefer financial investment or share repurchase unless it wants to engage in empire-building.\textsuperscript{15}

The cases in which no positive-NPV project is found and management prefers financial investment are consistent with Jensen’s (1986) assertion that management will often hoard cash rather than pay dividends. As he notes, this problem must be addressed by finding ways to induce management to disgorge the cash into more efficient uses. But Jensen’s claim is not focused on the fact that by hoarding cash, management benefits from its options. Indeed options have been considered as a means of getting management to engage in more capital investment. But as seen here, options might not achieve this desired result.

In this section we have examined what happens when the firm considers alternative uses of funds. We observe that options create conflicts between management and shareholders. Another possible scenario is that the firm has an inadequate supply of internal financing but access to capital markets. In the appendix we examine the cases of where external debt and equity fund the capital investment and also the case where the required outlay for capital investment is less than the available cash. None of our conclusions change.

\textbf{E. On the Assumption of }$\gamma < 1$ \textit{in a Principal-Agent World}

These results have been derived under extremely mild assumptions. In this section we show that the only critical requirement, that $\gamma < 1$, is supported in a principal-agent world. Let us assume that, in accordance with the results in Table 2, both principal and agent agree that finding a positive-NPV project is the best result for both parties. The second best result for management is financial investment, while the second best result for the shareholders is ordinary dividends. We assume that management controls the decision of what to do with the cash so if it does not find a positive-NPV project, it will engage in financial investment. Formally, we assume that there is a probability $p(\gamma)$ that management will find a positive-NPV project with complementary probability $1 - p(\gamma)$ that management will have the firm engage in financial investment. We assume that the number of options awarded increases the likelihood of management finding a positive-NPV project but does so at a decreasing rate. Thus,

$$p'(\gamma) > 0$$

$$p''(\gamma) < 0.$$\textsuperscript{15}

\textsuperscript{15}Management might possibly undertake a negative-NPV project if that project enables management to increase its power or secure an entrenched position.
The value of the firm is, therefore,

\[ V_0(\gamma) = \left\{ \frac{p(\gamma)[(1-\gamma)C_1 + \gamma C_r^{-1} + \gamma C_0 + (1-\gamma)\Delta C_1]}{+(1-p(\gamma))[\gamma C_r^{-1} + \gamma C_0 + (1-\gamma)C_0 r]} \right\} r^{-1} \]

The board of directors maximizes the value of the firm by choosing the optimal number of options:

\[ \max_\gamma V_0(\gamma). \]

Differentiating, we obtain

\[ V_0(\gamma)' = r^{-1} \left\{ \frac{p(\gamma)'[(1-\gamma)C_1 + \gamma C_r^{-1} + \gamma C_0 + (1-\gamma)\Delta C_1]}{-p(\gamma)'[(1-\gamma)C_1 + \gamma C_r^{-1} + \gamma C_0 + (1-\gamma)C_0 r]} + p(\gamma)[-C_1 + C_0 r^{-1} + C_0 - \Delta C_1] + (1-p(\gamma))[-C_1 + C_0 r^{-1} + C_0 - C_0 r] \right\}. \]

Cancelling and rearranging gives the simplified expression

\[ V_0(\gamma)' = r^{-1} \left\{ \theta + \frac{p(\gamma)[C_0 r - \Delta C_1]}{p(\gamma)'(\Delta C_1 - C_0 r)} \right\}, \]

where \( \theta = -C_1 + C_0 r^{-1} + C_0 - \Delta C_1 \), which is negative. The solution is

\[ \gamma = 1 + \frac{\theta + p(\gamma)(C_0 r - \Delta C_1)}{p(\gamma)'(\Delta C_1 - C_0 r)}. \]

The sign of each term assures us that \( \gamma < 1 \), and the second-order condition identifies the result as a maximum. Hence, if the board acts in the shareholders’ interests and recognizes that management will either invest the cash in a positive-NPV project and if one is not available, it will invest the cash in securities, the number of options will be less than the number of shares. Thus, our results, while quite general without modeling a principal-agent relationship, are still upheld under the mildly restrictive conditions of a simple principal-agent model.

IV. Solving the Problems

The problems described so far arise from the fact that management shares in the benefits of projects that existed before the options were awarded. An ideal incentive system would reward management only when it adds value. Hence, the options should have value only when a positive-NPV project is found. We now look at possible ways to solve this problem.

A. New Capital Investment Criterion

Let us first attempt to address the problem by defining a new investment decision criterion so that incentive payments are taken into account before committing the funds. Define an acceptable project as one in which the \( NPV \) must exceed the cost
of the options. We refer to this as the adjusted-NPV, and it amounts to deducting the cost of the options from the net present value. Such a rule would eliminate outcomes in which management accepts positive-NPV projects and the shareholders are worse off because the cost of the options exceeds the NPV. But this approach is not the best solution. Management would still receive value as a result of cash flows that existed before the options were awarded. In addition, the firm will have to reject projects that have positive conventional NPV, because the cost of the options exceeds the conventional NPV.\textsuperscript{16}

\textbf{B. An Option on the New Project’s Cash Flow}

Technically it would be possible to create an option that pays off based only on the new project’s cash flow. As a practical matter, such an option would be difficult to construct, as it would be hard to separate the component of the stock price associated with the project’s cash flow from the component based on the existing cash flows. As we show next, there is an easily implementable solution that will enable the firm to accept all projects when conventional NPV is positive and have management capture only a portion of the gain to the shareholders.

\textbf{C. Adjusting the Exercise Price}

A solution lies in adjusting the option exercise price. Instead of offering options with an exercise price equal to the current stock price, we let the exercise price be indexed to the firm’s cost of capital, and, thus, set at \( S_0 r \). From Equation (1), the strike now becomes \( C_0 r + C_1 \). We now show that all of the problems identified above with conventional options disappear.

First we show that the options have value if and only if the traditional NPV is positive. Recall that the options have value if \( C_1 + \Delta C_1 > X \) where \( X \) is the strike, which is now set to \( S_0 r \):

\[
C_1 + \Delta C_1 > S_0 r \\
C_1 + \Delta C_1 > C_0 r + C_1 \\
\Delta C_1 r^{-1} > C_0,
\]

which is the positive NPV condition. Next we show that the option value cannot exceed the NPV. We specify that the option value is less than NPV and proceed algebraically:

\textsuperscript{16}Of course, if the executive has already been awarded the option, shareholders would want a positive-NPV project to be accepted because the option payoff is a sunk cost.
\[\gamma(C_i + \Delta C_i - S_0 r)r^{-1} < \Delta C_i r^{-1} - C_0\]
\[\gamma < \frac{\Delta C_i r^{-1} - C_0}{C_i r^{-1} + \Delta C_i r^{-1} - S_0}\]
\[\gamma < \frac{\Delta C_i r^{-1} - C_0}{C_i r^{-1} + \Delta C_i r^{-1} - (C_i r^{-1} + C_0)}\]
\[\gamma < 1.\]

Thus, our constraint that no more options are awarded than shares of stock outstanding is sufficient to ensure that the cost of the options does not exceed the NPV. In effect, positive-NPV projects result in a sharing of the NPV with management. The conventional NPV decision criterion can be safely used, though the increase in shareholder wealth is not the NPV, but instead is the NPV after deducting the cost of the managerial incentives.

Now let us consider how these options work when management cannot find a positive-NPV project. Suppose \(C_0\) is used to pay a dividend. Then the stock price drops to \(C_i r\). The strike is still based on the cum-dividend stock price, \(S_0 = C_0 + C_i r\), and, thus, is \(S_0 r = C_0 r + C_i\). The value of the option at time 1 will be \(\text{Max}(0, C_i - (C_0 r + C_i)) = 0\). The option is then sure to expire out-of-the-money, so its value at time 0 is zero. A similar result is obtained for the case of share repurchase. For financial investment, the NPV is zero and the option expires precisely at-the-money and, therefore, with no value.

**D. Shares Instead of Options**

Many firms use shares with or without options to align the interests of management and shareholders. Shares have the feature that management loses dollar for dollar if the firm’s performance is poor, while options result in a truncated loss. Shares can be viewed as options with zero exercise price. The cash flow at time 1 is \(C_i + \Delta C_i\) and, with shares, management receives \(\gamma S_i = \gamma(C_i + \Delta C_i)\). Thus, management receives a payoff provided the firm has value at time 1, that is, \(S_i > 0\), which is guaranteed by the absence of leverage. It is easy to show that the cost can exceed the NPV and that management can generate value for itself with NPV < 0. As with options, management receives a portion of the cash flows that were in place before the options were granted. With shares instead of options, it is not possible to solve these problems because the effective strike price is zero. That is, the shares cannot be indexed to the cost of capital, because there is no exercise price.

**E. Cost-of-Capital Indexed Options**
Options in which the exercise price is indexed to the cost of capital have been mentioned previously in the literature but not explored in an analytical framework. Jensen (2001) discusses them, referring to a 1990 paper by Stewart (1990). As Jensen argues, these options are better for executives who are able to create shareholder wealth and, hence, should be desired by executives that a firm would want to retain and undesired by executives that the firm would not want.

Modifying Jensen’s example slightly, we note that a company that issues an at-the-money option with 10 years to maturity can see its management quite enriched, while shareholder wealth decreases. Interestingly, noted investment legend Benjamin Graham may well have recognized the problem with at-the-money options instruments in his classic work *The Intelligent Investor* in 1949.

> Excessive compensation to officers is by no means a negligible matter. There are real abuses here, especially through the use of stock options at inadequate prices … (p. 208).

Vanguard founder John Bogle (2005) also recently speaks of this problem:

> [T]he fixed-price stock option is fundamentally flawed as a method of aligning the interests of ownership and management: They are not adjusted for the cost of capital, providing a free ride even for executives who produce only humdrum returns. (p. 16)

The analysis so far has considered options only in a world of cash flow certainty. Uncertainty is captured by the question of whether management can find value-increasing projects. We now look at how cash flow uncertainty affects our analysis.

**F. The Effect of Uncertainty**

With uncertain cash flows, we specify that the firm continues to have current cash of $C_0$ and expected future cash of $E(C_1)$. The current value of the firm is $S_0 = C_0 + E(C_1)/k$ where $k$ is one plus the appropriate risk-adjusted discount rate. The manager can invest $C_0$ into a risky project offering an expected payoff of $E(\Delta C_1)$. Assuming the project has the same risk as the company’s current projects, the NPV is $E(\Delta C_1)/k - C_0$. The cost-of-capital indexed option will have a strike of

$$X = kS_0 = k(C_0 + E(C_1)/k) = C_0k + E(C_1).$$
When the option expires, the realized cash flows will be \( C_1 \) and \( \Delta C_1 \). Thus, the payoff of a cost-of-capital indexed option is

\[
Max(0, C_1 + \Delta C_1 - X) = Max(0, C_1 + \Delta C_1 - (C_0k + E(C_1))) \\
= Max(0, C_1 - E(C_1) + \Delta C_1 - C_0k).
\]

Thus, whether the option pays off depends on a combination of the performance of the existing project relative to expectations and the performance of the new project relative to its time-adjusted initial outlay. The option’s payoff can be positive if the existing projects perform well and the new project performs poorly or vice versa. Thus, it would appear that management can benefit from projects currently in place, a problem we wanted to avoid. Moreover, management might not benefit even if the new project it initiates performs well.

Uncertainty, however, illustrates another reasonable means by which incentivized management can create value: by managing existing and new projects in such a way as to produce outcomes that exceed expectations. Consider a firm with no cash and no investment opportunities. Thus, its value is simply \( E(C_1)/k \). It would appear to be inadvisable to grant options to management. If, however, management can cause the existing projects to pay off more than \( E(C_1) \), shareholder wealth will increase. If cost-of-capital adjusted options are used, the strike price would be \( E(C_1) \), and the option would pay off \( Max(0, C_1 - E(C_1)) \). Thus, if management is successful in causing the firm’s existing projects to beat expectations, the option will pay off. If not, the option will expire worthless. Thus, under uncertainty we see that options still provide the proper incentive even if management has no cash to invest and no positive-NPV projects. Options can induce management to take actions that create more favorable payoffs from existing projects. Note, however, that if conventional options are used, the payoff will be \( Max(0, C_1 - E(C_1)/k) \), and management does not need to completely beat expectations. It need only produce \( C_1 > E(C_1)/k \).

As noted in the beginning of this section, if there is cash available to invest, the option payoff will be \( Max(0, C_1 - E(C_1) + \Delta C_1 - C_0k) \). In this case, management is rewarded by the sum total of its performance in managing both the existing projects and new projects. The option payoff is driven by the combined total of the value created by managing existing projects, whereby \( C_1 \) is benchmarked to \( E(C_1) \), and the value created by identifying and managing new projects, whereby \( \Delta C_1 \) is benchmarked to \( C_0k \). It is entirely possible that, given limited resources, management could choose to neglect either the old or new projects in favor of the other, but this choice causes no problem. It is the performance of the total portfolio of projects that matters.
Whether the company has cash or not, the cost-of-capital indexed option will always have value when awarded because there is a non-zero probability that it will expire in-the-money. Management cannot, however, capture that value at time 0 due to illiquidity, vesting requirements, and exercise restrictions. In practice, American-style options that vest before expiration can cause problems, however, because management might be able to capture value from the options before it has been determined that management has been successful in creating value for shareholders. We explore these issues in the next sub-section.

G. Uncertainty and Early Exercise

Uncertainty raises interesting questions about how an early exercise feature would work. Because we have already shown that cost-of-capital indexed options can be useful incentive devices even when there are no new investment opportunities, we simplify the scenario by removing the time 0 cash balance of $C_0$. Now, we give the problem a two-period horizon such that the current stock price is $S_0 = E(C_1)/k + E(C_2)/k^2$. Let the firm award management a two-period cost-of-capital indexed option, vested and exercisable at time 1. Now move forward to time 1, where $C_1$ is the realized cash flow. The exercise value of the option is $\text{Max}(0, C_1 + E(C_2)/k - (E(C_1)/k + E(C_2)/k^2)k) = \text{Max}(0, C_1 - E(C_1))$. If management can cause the firm to exceed expectations in time 1, early exercise would lead to capture of value for management, though early exercise would not necessarily be the optimal strategy. Now move forward to time 2 and assume that the time 1 cash flow is invested at the cost of capital, which can always be done with financial investment. The exercise value at time 2 would be $\text{Max}(0, C_2 + C_1 k - E(C_1)k - E(C_2))$.

These results show that if management can cause the existing projects to beat expectations in time 1 but not at time 2, it benefits from exercising at time 1. Of course, it does not know if it can beat expectations at time 2. Hence, it might exercise if it believes that the time 2 performance will fall below expectations. If it can cause the existing projects to beat expectations at time 2 but not at time 1, it can benefit only by exercising at time 2. If it waits until time 2 to exercise, it must beat expectations at time 2 by an amount sufficient to overcome any underperformance at time 1. If it can beat expectations in both periods, management will benefit by waiting to exercise at time 2. Of course, if it cannot beat expectations in either period, it will not exercise at
all. Whether management exercises at time 1, however, depends not only on its expectations for time 2 but its risk tolerance.\footnote{It would be a simple matter to incorporate a revision at time 1 of the time 2 cash flow expectation, but the essential point of this example would be the same.}

If management can invest $C_i$ at a rate higher than the cost of capital, it would be more inclined to not exercise at time 1. Thus, as a general rule, we see that management will exercise early when it expects poor performance and will not exercise early when it expects good performance.\footnote{It should be noted that this same type of analysis could be conducted for any investor holding an option. As is well known, in the absence of dividends ordinary call options are never exercisable prior to expiration. Hence, this analysis could seem inconsistent with standard option theory. Keep in mind, however, that holders of ordinary options can always sell options at a price at least as high as the exercise value. For executive stock options, exercise is the only way to realize cash.} This statement is true regardless of whether the option is indexed or not. Indexing imposes a higher hurdle for management’s exercise decision, a hurdle that does a better job of rewarding management when shareholders benefit and not rewarding management when shareholders do not benefit.

What do these results suggest about how firms should set their vesting and early exercise periods? Early exercise removes the incentive for management beyond the first period. It allows management to bail out when it thinks that it cannot cause the firm to exceed expectations. One could make a case, therefore, that early exercise is an undesirable feature, but its inclusion in many contracts is a concession to management to make employment with the firm more attractive. Early exercise grants some liquidity to an instrument that is otherwise illiquid.

\section*{H. Uncertainty, Luck, and Skill}

Under uncertainty we showed that management benefits whenever the firm’s existing projects outperform expectations and when its new projects produce more cash than the opportunity cost. These results can occur from factors outside management’s control as well as from factors within management’s control. Factors outside management’s control include the effect of unexpected market and industry performance, as well as random noise. The role of luck in managerial compensation has been examined by Bertrand and Mullainathan (2001) and Garvey and Milbourn (2006), both papers of which confirm that compensation is greatly influenced by systematic luck. Garvey and Milbourn in fact find that the effects are one-way, resulting in positive benefits from systematic luck without negative consequences. Let us adjust the cost of capital indexed option model to remove the effect of luck.

Illustrating with the simple case of no cash available at time 0, the payoff of the option at time 1 is $\text{Max}(0, C_1 - (E(C_1)/k)k) = \text{Max}(0, C_1 - E(C_1))$. We wish to adjust this
payoff so that the unexpected influence of the market is removed. The stock price at
time 1 is the expected stock price plus the unexpected stock price:
\[ S_1 = E(S_1) + \omega(S_1), \]  
(4) where \( \omega(S_1) \) is the unexpected portion of the stock price. Dividing by \( S_0 \) we obtain
\[ \frac{S_1}{S_0} = \frac{E(S_1)}{S_0} + \frac{\omega(S_1)}{S_0}. \]
The second expression on the right-hand side is the unexpected return on the stock. Its
numerator consists of
\[ \omega(S_1) = \omega(M) + \omega(E) \]
where \( \omega(M) \) is the unexpected performance of the stock that is attributed to the market
and \( \omega(E) \) is the unexpected performance of the stock that is attributed to the
performance of the executive.

We decompose the stock price into three components:
\[ S_1 = E(S_1) + \omega(M) + \omega(E) \]
We want to remove the effect of the unexpected performance of the market and the
expected performance of the stock. We propose to adjust the exercise price in the
following manner:
\[ X = S_0(k + (r_m - \mu_m)\beta) \]
(5) where \( r_m \) is one plus the return on the market index, \( \mu_m \) is the expected return on the
market, and \( \beta \) is the systematic risk. Note that
\[ S_0k = E(C_1) \]
\[ S_0(r_m - \mu_m)\beta = \omega(M). \]
The first expression simply restates that the current stock price times the cost of capital
is the expected future cash flow. The second expression defines that the unexpected
effect of the market is the stock price times the unexpected return on the market times
the beta.

Now we see that the payoff of the cost-of-capital-indexed option is
\[ \text{Max}(0, C_1 - X) = \text{Max}(0, C_1 - (S_0k + (r_m - \mu_m)\beta)) \]
\[ = \text{Max}(0, C_1 - S_0k - (r_m - \mu_m)\beta)) \]
\[ = \text{Max}(0, C_1 - E(C_1) - \omega(M)) \]
(6) We see that the firm’s performance must not only exceed expectations but it must do so
by more than the unexpected performance of the market.

Now note that since \( k = r + (\mu_m - r)\beta \), we substitute as follows:
\[
X = S_0(k + (r_m - \mu_m)\beta) \\
= S_0(r + (\mu_m - r)\beta + (r_m - \mu_m)\beta) \\
= S_0(r + (r_m - r)\beta)
\] (7)

In other words, to remove the portion of the firm’s performance that is determined by the unexpected performance of the market, the strike should be set at the current stock price increased by the ex post capital asset pricing model. This is the standard adjustment used in determining alphas for investment managers. Option grants should employ a similar hurdle. Management should be rewarded only if it increases value for shareholders after accounting for risk and luck, as determined by the unexpected performance of the market. As noted, an industry factor could also be added.

I. Cost-of-Capital Indexed Options versus Market-Indexed Options

Cost-of-capital indexed options appear to be similar to market-indexed options, which have been examined by Johnson and Tian (2000). In their model, the exercise price is adjusted to equal the expected stock price, conditional on the realized level of the index and zero unsystematic risk. Jin (2002) and Garvey and Milbourn (2003), however, argue that many CEO’s can effectively hedge away the market, thereby obviating the need for market-indexed options. Our paper has explored the justification for cost-of-capital indexed options. We show that these options correct the problems created by standard options, which reward managers for performance that existed before the options were granted. Further research would be necessary to determine if market-indexed options solve such problems.

J. Risk Shifting

Recall that with the cost-of-capital indexed option under uncertainty, we assume that a new project accepted by management has the same risk as the existing project. This assumption is critical and if it is violated, we have introduced a third source of uncertainty.\(^\text{19}\) Recall that the payoff of the cost-of-capital indexed option under uncertainty is

\[
\text{Max}(0,C_1 + \Delta C_1 - X) = \text{Max}(0,C_1 - E(C_1) + \Delta C_1 - C_0 k).
\]

If the new project has risk different from that of the existing projects, then its cost of capital will not equal \(k\). Yet the option payoff treats the opportunity cost of cash invested as \(k\). Let us assume the new project has a cost of capital of \(k'\). Thus, when the options are awarded, the shareholders bear the risk that management will shift the risk of the firm. Such an action can indeed be desirable, as often options are awarded to

\(^{19}\)Recall that the first source of uncertainty is whether management will find a positive-\(NPV\) project. The second source of uncertainty is the cash flows from existing and new projects.
induce management to take more risks. The cost-of-capital adjustment to the option strike, however, cannot properly reflect the new risk, because the risk is unknown when the option is awarded.

One solution to the problem is for the option contract to stipulate the risk that management can take. This solution would, however, force management to reject positive-\textit{NPV} projects from restricted risk classes. A better solution is that the option be granted such that if there is a subsequent change in risk, the strike is adjusted accordingly. Let \( k' \) be the adjusted cost-of-capital, which is given as follows:

\[
k' = \left( \frac{1}{S_0} \right) (E(C_1) + C_0 k').
\]

Then the option payoff will be:

\[
\text{Max}(0, C_1 + \Delta C_1 - X) = \text{Max}(0, C_1 + \Delta C_1 - S_0 k')
\]
\[
= \text{Max}(0, C_1 + \Delta C_1 - S_0 (1/S_0) (E(C_1) + C_0 k'))
\]
\[
= \text{Max}(0, C_1 + \Delta C_1 - (E(C_1) + C_0 k'))
\]
\[
= \text{Max}(0, C_1 - E(C_1) + \Delta C_1 - C_0 k').
\]

Here we see that the proper cost of capital is applied to the capital investment of \( C_0 \). Management and the shareholders still benefit if the project in place and the new project exceed expectations.

For a standard option, risk shifting is an even greater problem. The payoff of the standard option is

\[
\text{Max}(0, C_1 - E(C_1) / k + \Delta C_1 - C_0),
\]

which means that there is no adjustment whatsoever for the cost of capital on the investment of \( C_0 \). Except for the rare case of the new project having an extremely low cost of capital, the cost-of-capital indexed option would be better, even if it were not adjusted to reflect the new cost of capital. But, of course, an adjustment for the different risk of the new project vis-à-vis the old is appropriate.

Because options are awarded to incentivize management, risk-shifting would not be unusual. Indeed we noted earlier that options are oftentimes used to induce management to take more risk. What we have seen from this model, however, is that not knowing the risk that management will take is another element of uncertainty. In standard financial theory, this risk poses no problem. As long as management uses the appropriate hurdle rate, a positive-\textit{NPV} project creates value when investors become informed about the new project. But if management has been granted options to induce it to take on new projects that might have different risks, these options must be designed to either permit an adjustment for the new risk or to contractually stipulate the risk.
that management can take. The latter will result in underinvestment. The former approach seems feasible and appropriate.\textsuperscript{20}

V. Empirical Estimates

We have demonstrated that if at-the-money options are awarded as incentives, then management will benefit from cash flows in place before the options are awarded. Thus, traditional options reward management for actions taken before the options are granted. If the options were indexed to the cost of capital, however, management would benefit only when shareholders benefit. These cost-of-capital-adjusted options are virtually non-existent, but we can estimate the difference this adjustment would make. If options are strictly viewed as incentive devices, we could consider this differential to represent an economic loss. If options are viewed as part compensation and part incentives, then this differential would represent the compensation component because it rewards management for cash flows in place before the options are awarded.

We also showed that executives can benefit from at-the-money options by choosing financial investment, or hoarding cash, over paying dividends. As noted earlier, empirical evidence elsewhere is consistent with the notion that options seem to create an aversion to dividends. In this section, we add to that discussion and provide empirical support for this model by examining whether the difference in value between a cost-of-capital indexed option and an original-issue at-the-money option is related to the firm’s cash position. We also saw earlier that executives can benefit from investments that destroy shareholder value. Although we showed that negative-NPV projects are inferior to financial investment and dividends for executives, there is no assurance that executives would not waste corporate resources investing in such projects. Indeed empire building and entrenchment are strong motivators for accepting such projects.\textsuperscript{21}

When coupled with the potential that stock options can pay off for projects that reduce shareholder wealth, executives may well engage in wasteful investment, or as it is termed, overinvestment. These effects can be empirically tested.

To examine these issues, we select a sample of firms, estimate their costs of capital, collect information about the stock options of their top executives, and estimate the difference in value if these options had been indexed to the cost of capital. We then compute the difference in value of at-the-money options versus cost-of-capital indexed options and relate that measure to free cash flow and overinvestment.

\textsuperscript{20}Management can also shift the risk after the project is in place, in which case the option contract would need to permit an adjustment at that time as well.

A. Data and Methodology

We use all non-financial firms with the required data available in Compustat, CRSP, and the ExecuComp data bases. ExecuComp provides annual data from 1992-2005 on the compensation and options of the top five executives. These constraints, along with the availability of Compustat and CRSP data, give us 4,553 firm-years. We estimate the cost of capital for each firm following a procedure used by Vassalou and Xing (2004), and similarly by Hillegeist et al (2004) and Bharath and Shumway (2004), which is based on an application of the Merton (1974) model of corporate debt and equity as options. We then revalue the outstanding options using the Black-Scholes-Merton model with the exercise price increased by the cost of capital. We then compute the difference in value of the cost-of-capital option versus the at-the-money option and divide that difference by the value of the equity. We also estimate free cash flow and overinvestment with a technique used by Richardson (2006), and regress these estimates on the option value difference and other variables that are known to be correlated with free cash flow and overinvestment.

B. Estimating the Cost of Capital

Following Merton (1974), we treat the equity as a call option on the assets with value

\[
V_E = V_A N(d_1) - B e^{-rT} N(d_2) = \ln \left( \frac{V_A}{B} \right) + (r + 0.5 \times \sigma_A^2) \tau - \frac{\sigma_A \sqrt{\tau}}{\sigma_A \sqrt{\tau}}, \quad d_2 = d_1 - \sigma_A \sqrt{\tau},
\]

where \( V_E \) is the market value of equity, \( B \) is the face value of debt, \( V_A \) is the market value of the assets, \( r \) is the risk-free rate, \( T \) is the maturity of the debt, and \( \sigma_A \) is the volatility of the assets. The procedure used by Vassalou and Xing and others starts with the assumption that the debt maturity is one year. The face value of the debt is the debt due within one year plus one-half of the long-term debt, and the risk-free rate is the one-month Treasury bill rate. We take an estimate of the volatility of the market value of the stock from the previous year as a starting point for estimating \( \sigma_A \). We then estimate the value of the assets, \( V_A \), in the above equation. This procedure is repeated for every trading day of the previous year, which generates a series of values of the assets, from which we can then estimate a volatility of the assets. We then insert this figure into the formula as the next estimate of \( \sigma_A \) and repeat the procedure until the difference in consecutive estimates is less than 0.0001. Then, the mean return of the
daily series of market values of the assets provides an estimate of the expected rate of
return, which is our proxy for the cost of capital.

C. Free Cash Flow, Overinvestment, and At-the-Money Options

We estimate free cash flow and overinvestment using a procedure developed by
Richardson (2006). First we estimate total investment as the sum of capital
expenditures, acquisition, and research and development, less the sale of property, plant,
and equipment. This figure can be decomposed into maintenance and new investment,
the former of which is available on COMPUSTAT. New investment can then be
decomposed into expected new investment and unexpected new investment, the latter of
which is the proxy for overinvestment. We then regress total investment on the previous
year’s measure of growth opportunities, leverage, cash, age of the firm, total assets, the
previous year’s stock return, and the previous year’s new investment. We also consider
two dummy variables, one that reflects each of the years 1992-2005 and one that
captures the two-digit SIC industry code. Growth opportunities are measured as the
value of assets in place relative to market value of equity. Total investment and cash
are scaled by total assets. The value of assets in place is estimated following a procedure
used by Richardson. The fitted total investment is the expected investment, and the
residual is the unexpected investment, with a positive residual indicative of
overinvestment. To estimate free cash flow, we need cash flow from assets in place,
which is the sum of operating cash flows and research and development less maintenance
expenditures and is scaled by total assets. Free cash flow is then the cash flow from
assets in place net of expected new investment.

Table 3 contains summary statistics of these variables. New investment and cash
flow from assets in place average about 9% of total assets, and the cash balance is about
16% of total assets. The average firm has book value of total assets of about $5.4 billion
and market value of about $4.9 billion.

First we replicate the Richardson results. We test three versions of the model,
each of which contains the same metric variables but differ with respect to year and
industry dummies. Model I contains neither dummy, model II contains an industry
dummy but not a year dummy, and Model III contains both dummies. The results are
not notably different across models. Size is not significant when both dummy variables
are used, is significant at the 5% level when only an industry dummy is used, and is not
significant when neither dummy is used. R²s vary from only from 32.2% to 34.4%,
figures that are similar to Richardson’s. Model III, which includes year and industry
dummies, is most consistent with Richardson’s results. Hence, we use that model to
estimate expected new investment for the purpose of estimating unexpected new investment, or overinvestment.

D. Empirical Results

As noted, we measure the option value difference by using the Black-Scholes-Merton model to value the options with the actual exercise prices and with the exercise prices increased by the cost of capital. We obtain option values using ExecuComp’s estimates of the risk-free rate with a maturity of seven years, the assumption made by ExecuComp based on the expectation of early exercise. Table 4 contains our estimates of the difference in value of the actual at-the-money options versus the hypothetical cost-of-capital indexed options expressed as a percentage of various measures of interest. The mean (median) option value difference is 0.27% (0.07%) of the market value of the equity, 0.33% (0.07%) of the book value of the assets, 127.8% (44.9%) of the salary as reported in ExecuComp, 73.9% (27.9%) of salary plus bonus, 17.9% (11.1%) of total compensation, which includes salary, bonus, restricted stock, Black-Scholes-Merton option value, long-term incentive payments, and any other compensation, and 42.1% (32.6%) of option value based on Black-Scholes-Merton.

We then regress free cash flow on the option value difference relative to market value and the variables as noted above, with and without year and industry dummies. Results are shown in Table 5. Model I, which does not include year and industry dummies, has an R² of 10.9% and Model II, which does include year and industry dummies, has an R² of 16.3%. In Model I, the option value difference is positively related to free cash flow and significant at the 10% level, and in Model II, the option value difference is positively related to free cash flow and significant at the 5% level. Models III and IV include only firm-years with positive free cash flow. Though the R²’s are lower, the option value difference is positively related to free cash flow and significant at the 1% level. Thus, the option value difference does appear to be positively and significantly related to free cash flow, as our model predicts.

We also wish to examine the relationship between the option value difference and overinvestment. Recall that, following Richardson (2006), we estimate overinvestment as the positive residual from a regression of investment on the variables previously discussed. Hence, we do not need to incorporate these variables again into the regression to explain overinvestment. Thus, we regress overinvestment only on the option value difference and free cash flow with and without the dummy variables for year and industry. Free cash flow (FCF) is captured with two dummy variables. The first is FCF when FCF > 0 and zero otherwise. The second is FCF when FCF < 0 and zero
otherwise. Splitting the variable in this manner allows us to capture an asymmetric response of overinvestment to free cash flow. Table 6 shows the results. Recall, however, that overinvestment is a positive residual from the regression of investment on the variables previously mentioned, while underinvestment is a negative residual. Initially we simply enter unexpected investment, both over- and underinvestment. We find that unexpected investment is significantly and positively related to the presence of positive free cash flow. The option value difference does not show as significant. Models III and IV, however, use only firms with positive unexpected investment, which is overinvestment. Here we find that the option value difference is highly and positively related to overinvestment.

Thus, the empirical results confirm the principal theoretical propositions from the model. The shareholders of firms with high levels of free cash flow and overinvestment are likely to have options that reward management more for past performance than do the shareholders of firms with low levels of free cash flow and overinvestment. Thus, the problems of free cash flow and overinvestment could be alleviated by indexing the options to the cost of capital.

VI. Conclusions

This paper shows that conventional at-the-money options do not provide a one-to-one mapping of managerial benefit to shareholder benefit. Under certain cash flows and uncertainty of investment opportunities, conventional options allow management to benefit from cash flows that existed before the options were awarded. As such, the cost of the options can exceed any value created by management, and management can benefit even when it destroys shareholder value. We show that indexing the option exercise price to the firm’s cost of capital solves these problems.

When we add the condition of cash flow uncertainty, we find that management can create value not only by finding positive-NPV projects but also by managing existing projects so that results exceed expectations. With conventional options, management can benefit if existing or new projects underperform expectations. When options are indexed to the cost of capital, management benefits only if outcomes exceed expectations. Thus, management can take on new positive-NPV projects that ultimately fail to meet expectations but can offset this result if it can manage existing projects so that they exceed expectations. It is the total value created by management after receipt of the options that determines whether the options work as incentives.

Empirical tests show that at-the-money options reward management more when there are high levels of free cash and flow and overinvestment. The average benefit to
management amounts to only about a quarter of a percent of the value of the equity but about 18 percent of total compensation and over 40 percent of option value. If options are viewed as incentives, this seems like a high cost. If options are viewed as part compensation-part incentives, these figures provide heretofore unknown estimates of how compensation and incentives comprise the value of an option.
Table 1. Managerial and Shareholder Benefits from Alternative Uses of Cash

$\gamma$ is the number of options awarded, $C_0$ is the cash flow at time 0, $C_1$ is the cash flow at time 1 from existing projects, $\Delta C_1$ is the cash flow at time 1 from the new project, and $r$ is the cost of capital. The benefits are computed as the gain in value to the respective party less any cost or cash paid out.

<table>
<thead>
<tr>
<th>Alternative Use</th>
<th>To Management</th>
<th>To Shareholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital investment</td>
<td>$\gamma(C_1 - C_1r^{-1} - C_0 + \Delta C_1)$</td>
<td>$(1 - \gamma)C_1 + \gamma C_1r^{-1} + \gamma C_0 + (1 - \gamma)\Delta C_1$</td>
</tr>
<tr>
<td>Dividend</td>
<td>$\gamma(C_1 - C_1r^{-1} - C_0)$</td>
<td>$(1 - \gamma)C_1 + \gamma C_1r^{-1} + C_0r + \gamma C_0$</td>
</tr>
<tr>
<td>Strike-adjusted dividend</td>
<td>$\gamma(C_1 - C_1r^{-1})$</td>
<td>$(1 - \gamma)C_1 + \gamma C_1r^{-1} + C_0r$</td>
</tr>
<tr>
<td>Share repurchase</td>
<td>$\gamma(C_1 - C_1r^{-1} - C_0 + C_0r)$</td>
<td>$(1 - \gamma)C_1 + \gamma C_1r^{-1} + \gamma C_0 + (1 - \gamma)C_0r$</td>
</tr>
<tr>
<td>Financial investment</td>
<td>$\gamma(C_1 - C_1r^{-1} - C_0 + C_0r)$</td>
<td>$(1 - \gamma)C_1 + \gamma C_1r^{-1} + \gamma C_0 + (1 - \gamma)C_0r$</td>
</tr>
</tbody>
</table>
Table 2. Order of Preference of Management and Shareholders for Each Use of Funds When Stock Options are Awarded

$\gamma$ is the number of options awarded, $C_0$ is the cash flow at time 0, $C_1$ is the cash flow at time 1 from existing projects, $\Delta C_1$ is the cash flow at time 1 from the new project, and $r$ is the cost of capital. CI is capital investment, FI is financial investment, which is investing in a zero-NPV project or correctly priced securities or assets, SR is share repurchased, D is dividends, and DSA is dividends with the strike reduced by the amount of the dividend.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Order of Preference for Management</th>
<th>Order of Preference for Shareholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $C_0 &lt; \Delta C_1 r^1$</td>
<td>CI &gt; FI ~ SR &gt; DSA &gt; D</td>
<td>CI &gt; D &gt; DSA &gt; SR ~ FI</td>
</tr>
<tr>
<td></td>
<td>(i) $(1 - \gamma) \Delta C_1 &gt; C_0 r$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(iii) $C_0 r - \gamma C_0 &gt; (1 - \gamma) \Delta C_1$</td>
<td></td>
</tr>
<tr>
<td>B. $C_0 &gt; \Delta C_1$</td>
<td>FI ~ SR &gt; DSA &gt; CI &gt; D</td>
<td>D &gt; DSA &gt; SR ~ FI &gt; CI</td>
</tr>
<tr>
<td>C. $C_0 = \Delta C_1$</td>
<td>FI ~ SR &gt; DSA ~ CI &gt; D</td>
<td>D &gt; DSA &gt; SR ~ FI &gt; CI</td>
</tr>
<tr>
<td>D. $\Delta C_1 r^1 &lt; C_0 &lt; \Delta C_1$</td>
<td>FI ~ SR &gt; CI &gt; DSA &gt; D</td>
<td>D &gt; DSA &gt; SR ~ FI &gt; CI</td>
</tr>
<tr>
<td>E. $C_0 = \Delta C_1 r^1$</td>
<td>FI ~ SR ~ CI &gt; DSA &gt; D</td>
<td>D &gt; DSA &gt; SR ~ FI &gt; CI</td>
</tr>
<tr>
<td>F. $\Delta C_1 = 0$</td>
<td>FI ~ SR &gt; DSA &gt; D ~ CI</td>
<td>D &gt; DSA &gt; SR ~ FI &gt; CI</td>
</tr>
<tr>
<td>G. $\Delta C_1 &lt; 0$</td>
<td>FI ~ SR &gt; DSA &gt; D ~ CI</td>
<td>D &gt; DSA &gt; SR ~ FI &gt; CI</td>
</tr>
</tbody>
</table>
Table 3. Summary Statistics

The data range is from 1992 to 2005 and all sample firms have data available in CRSP, Compustat, and ExecuComp databases. INV$_{New}$ is the total new investment, CF$_{AIP}$ is the cash flow generated from assets in place, and CASH is the balance of cash and short term investment. V/P is measure of growth opportunities, which is the ratio of the value of assets in place to the market value of equity. LEV is the debt ratio of book value of short- and long-term debt to total assets, AGE is the log of the number of years the firm has been listed in CRSP, SIZE is the log of total assets, and SR is the annual stock return defined as the log of the ratio of the market value of equity to its value of the previous year. TA is the book value of total assets and MV is the beginning balance of the market value of total equity. INV$_{New}$, CF$_{AIP}$, and CASH are scaled by total assets. There are 4,553 firm-years in the final sample.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>P(10th)</th>
<th>P(90th)</th>
</tr>
</thead>
<tbody>
<tr>
<td>INV$_{New}$</td>
<td>0.09309</td>
<td>0.06864</td>
<td>0.10827</td>
<td>-0.00125</td>
<td>0.21826</td>
</tr>
<tr>
<td>CF$_{AIP}$</td>
<td>0.09152</td>
<td>0.08346</td>
<td>0.10909</td>
<td>-0.01073</td>
<td>0.21569</td>
</tr>
<tr>
<td>CASH</td>
<td>0.16030</td>
<td>0.07848</td>
<td>0.19156</td>
<td>0.00923</td>
<td>0.44165</td>
</tr>
<tr>
<td>V/P</td>
<td>0.47826</td>
<td>0.42702</td>
<td>0.37202</td>
<td>0.14680</td>
<td>0.86202</td>
</tr>
<tr>
<td>LEV</td>
<td>0.21470</td>
<td>0.19478</td>
<td>0.17569</td>
<td>0.01286</td>
<td>0.42465</td>
</tr>
<tr>
<td>AGE</td>
<td>2.71441</td>
<td>2.70805</td>
<td>0.87915</td>
<td>1.60944</td>
<td>3.87120</td>
</tr>
<tr>
<td>SIZE</td>
<td>6.84745</td>
<td>6.70912</td>
<td>1.56160</td>
<td>4.98193</td>
<td>8.88907</td>
</tr>
<tr>
<td>SR</td>
<td>0.13142</td>
<td>0.13231</td>
<td>0.57146</td>
<td>-0.49707</td>
<td>0.76418</td>
</tr>
<tr>
<td>TA(millions)</td>
<td>5411.62</td>
<td>819.849</td>
<td>29060.16</td>
<td>145.756</td>
<td>7252.3</td>
</tr>
<tr>
<td>MV(millions)</td>
<td>4866.70</td>
<td>894.658</td>
<td>19934.04</td>
<td>170.847</td>
<td>8587.6</td>
</tr>
</tbody>
</table>
Table 4. Difference in Value between At-the-Money Options and Cost-of-Capital Options

OPTVALDIFF is the difference in option value between traditional at-the-money and cost-of-capital indexed options multiplied by the number of options granted. We measure the percentage of difference with respect to different valuables. TA is the book value of total assets and MV is the beginning balance of the market value of total equity. SAL is the dollar value of the base salary, SPB is the dollar value of the base salary plus bonus, and TOTC is the total compensation including base salary, bonus, restricted stock, Black-Scholes-Merton value of stock options, long-term incentive payouts, and all other total. OPTV is the stock options valued using S&P’s Black-Scholes-Merton methodology.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>P(10th)</th>
<th>P(90th)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPVALDIFF/MV</td>
<td>0.00269</td>
<td>0.00067</td>
<td>0.00833</td>
<td>0.00007</td>
<td>0.00605</td>
</tr>
<tr>
<td>OPVALDIFF/TA</td>
<td>0.00334</td>
<td>0.00074</td>
<td>0.00953</td>
<td>0.00008</td>
<td>0.00773</td>
</tr>
<tr>
<td>OPVALDIFF/SAL</td>
<td>1.27801</td>
<td>0.44903</td>
<td>4.60768</td>
<td>0.06051</td>
<td>2.68217</td>
</tr>
<tr>
<td>OPVALDIFF/SPB</td>
<td>0.73910</td>
<td>0.27872</td>
<td>2.15346</td>
<td>0.04233</td>
<td>1.58037</td>
</tr>
<tr>
<td>OPVALDIFF/TOTC</td>
<td>0.17857</td>
<td>0.11194</td>
<td>0.18057</td>
<td>0.01796</td>
<td>0.43737</td>
</tr>
<tr>
<td>OPVALDIFF/OPTV</td>
<td>0.42089</td>
<td>0.32646</td>
<td>0.34493</td>
<td>0.04259</td>
<td>0.93737</td>
</tr>
</tbody>
</table>
Table 5. Regression of Free Cash Flow on Option Value Difference

The dependent variable is free cash flow. The data range is from 1992 to 2005 and all sample firms have data available in CRSP, Compustat, and ExecuComp databases. OPTVALDIFF is the difference in option value between traditional at-the-money and cost-of-capital indexed options multiplied by the number of options granted. MV is the beginning balance of the market value of total equity. INVNew is the total new investment, CFAP is the cash flow generated from assets in place, and CASH is the balance of cash and short term investment. V/P is the measure of growth opportunities, which is the ratio of the value of assets in place to the market value of equity. LEV is the ratio of book value of short- and long-term debt to total assets, AGE is the log of the number of years the firm has been listed in CRSP, SIZE is the log of total assets, and SR is the annual stock return defined as the log of the ratio of current market value of equity to its value of the previous year. TA is the book value of total assets in millions. INVNew, CFAP, and CASH are scaled by total assets. Year dummies reflect each of the years 1992-2005, and the industry dummies capture two-digit SIC codes. There are 4,553 firm-years in the final sample and 2,468 firm-years with positive free cash flow, FCF>0. The numbers in parentheses are t-statistics based on heteroskedasticity-consistent standard errors. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% levels respectively.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FCF&gt;0</td>
<td></td>
<td>FCF&gt;0</td>
<td></td>
</tr>
<tr>
<td>OPTVALDIFF/MV</td>
<td>0.448</td>
<td>0.495</td>
<td>0.670</td>
<td>0.637</td>
</tr>
<tr>
<td>V/P_{t-1}</td>
<td>0.005</td>
<td>0.017</td>
<td>-0.019</td>
<td>-0.016</td>
</tr>
<tr>
<td>LEV_{t-1}</td>
<td>-0.076</td>
<td>-0.078</td>
<td>0.015</td>
<td>0.010</td>
</tr>
<tr>
<td>CASH_{t-1}</td>
<td>-0.076</td>
<td>-0.114</td>
<td>0.051</td>
<td>0.031</td>
</tr>
<tr>
<td>AGE_{t-1}</td>
<td>0.004</td>
<td>0.003</td>
<td>0.001</td>
<td>0.0001</td>
</tr>
<tr>
<td>SIZE_{t-1}</td>
<td>0.010</td>
<td>0.009</td>
<td>-0.003</td>
<td>-0.002</td>
</tr>
<tr>
<td>SR_{t-1}</td>
<td>0.009</td>
<td>0.013</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>INVNew_{t-1}</td>
<td>-1.154</td>
<td>-0.178</td>
<td>0.084</td>
<td>0.070</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.037</td>
<td>-0.034</td>
<td>0.074</td>
<td>0.067</td>
</tr>
<tr>
<td>Year dummies</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry dummies</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.109</td>
<td>0.163</td>
<td>0.079</td>
<td>0.094</td>
</tr>
</tbody>
</table>
Table 6. Regression of Unexpected Investment on Option Value Difference

The dependent variable is unexpected investment. The data range is from 1992 to 2005 and all sample firms have data available in CRSP, Compustat, and ExecuComp databases. OPTVALDIFF the difference in option value between traditional at-the-money and cost-of-capital indexed options multiplied by the number of options granted. MV is the market value of equity at the end of the previous year. FCF>0 is free cash flow when it is positive and zero otherwise. FCF<0 is free cash flow when it is negative and zero otherwise. Models III and IV use only observations in which there is overinvestment, defined as positive unexpected investment. Year dummies reflect each of the years 1992-2005, and the industry dummies capture two-digit SIC codes. There are 4,553 firm-years in the final sample and 859 firm-years with positive overinvestment. The numbers in parentheses are t-statistics based on heteroskedasticity-consistent standard errors. The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% levels respectively.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III (Overinvestment)</th>
<th>Model IV (Overinvestment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTVALDIFF/MV</td>
<td>0.168</td>
<td>0.156</td>
<td>0.926</td>
<td>0.925</td>
</tr>
<tr>
<td></td>
<td>(0.83)</td>
<td>(1.76)</td>
<td>(2.09)**</td>
<td>(2.03)**</td>
</tr>
<tr>
<td>FCF&gt;0</td>
<td>0.245</td>
<td>0.255</td>
<td>0.225</td>
<td>0.179</td>
</tr>
<tr>
<td></td>
<td>(5.26)**</td>
<td>(5.24)**</td>
<td>(3.64)**</td>
<td>(2.82)**</td>
</tr>
<tr>
<td>FCF&lt;0</td>
<td>0.085</td>
<td>0.090</td>
<td>-0.213</td>
<td>-0.178</td>
</tr>
<tr>
<td></td>
<td>(1.83)*</td>
<td>(1.85)*</td>
<td>(-4.20)**</td>
<td>(-3.64)**</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.006</td>
<td>-0.007</td>
<td>0.049</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(-2.22)**</td>
<td>(-1.23)</td>
<td>(14.45)**</td>
<td>(3.30)**</td>
</tr>
<tr>
<td>Year dummies</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry dummies</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.040</td>
<td>0.042</td>
<td>0.061</td>
<td>0.088</td>
</tr>
</tbody>
</table>
Figure 1. Management and Shareholders’ Decision Rule

This figures illustrates the relationship between NPV and the cash flow in place, $C_1$. It shows that when cash flow in place is sufficiently large, management can benefit from negative-NPV projects as well as from positive-NPV projects.
Appendix: Proofs of Propositions

Proof of Proposition 1:

A sufficient condition for the options to have value at time 0 is that the option be exercised at time 1 for certain, that is, \( C_1 + \Delta C_1 > S_0 \). If \( NPV \) is positive, we have \( \Delta C_1 r^1 > C_0 \). Since \( S_0 = C_0 + C_1 r^1 \), it follows that

\[
\Delta C_1 r^{-1} > S_0 - C_1 r^{-1}
\]

\[
\Rightarrow \Delta C_1 r^{-1} + C_1 r^{-1} > S_0.
\]

Because the terms on the left-hand side are discounted, clearly their undiscounted values exceed the right-hand side, \( S_0 \). Thus, \( C_1 + \Delta C_1 > S_0 \), so we know that the option will have value if management is successful in finding a positive-\( NPV \) project.

Proof of Proposition 2:

Assuming exercise, the cost of the options is \( \gamma (C_1 + \Delta C_1 - S_0) r^1 \), while net present value is, of course, \( \Delta C_1 r^1 - C_0 \). The condition in which the option cost exceeds the \( NPV \) is

\[
\gamma (C_1 + \Delta C_1 - S_0) r^1 > \Delta C_1 r^{-1} - C_0
\]

\[
\Rightarrow \gamma (C_1 + \Delta C_1 - S_0) > \Delta C_1 - C_0 r
\]

\[
\Rightarrow \gamma > \frac{\Delta C_1 - C_0 r}{C_1 + \Delta C_1 - S_0}
\]

Since \( \gamma \) is constrained to a value of between 0 and 1, we need only demonstrate that the right-hand side of the inequality is less than 1:

\[
\frac{\Delta C_1 - C_0 r}{C_1 + \Delta C_1 - S_0} < 1
\]

\[
\Rightarrow \Delta C_1 - C_0 r < C_1 + \Delta C_1 - (C_0 + C_1 r^{-1})
\]

\[
\Rightarrow \Delta C_1 - C_0 r < C_1 + \Delta C_1 - C_0 - C_1 r^{-1}
\]

\[
\Rightarrow C_0 - C_0 r < C_1 - C_1 r^{-1}
\]

The left-hand side is clearly negative, while the right-hand side is clearly positive. Thus, the critical ratio is less than 1. So the value of the options can exceed the \( NPV \) if \( \gamma \) exceeds the indicated ratio above, which itself is less than 1. Also, note that the maximum number of options for cost to not exceed \( NPV \) is a function of \( NPV \). Thus, it is not possible for the board to set \( \gamma \) such that the cost is guaranteed to not exceed \( NPV \).

Proof of Proposition 3:

It is a simple matter to rearrange the formula for the value of the options:
The first term in parentheses is clearly positive. The second term can be positive even if the NPV is negative. In other words, the undiscounted cash flow can exceed the initial outlay, even though the discounted cash flow does not exceed the initial outlay.

\[
\gamma(C_i + \Delta C_i - S_o)r^{-1} > 0 \\
C_i + \Delta C_i - S_o > 0 \\
C_i + \Delta C_i - C_o - C_i r^{-1} > 0 \\
(C_i - C_i r^{-1}) + (\Delta C_i - C_o) > 0
\]

Proof of Proposition 4:

Proposition 2 stated that the value of the options at expiration is

\[
(C_i - C_i r^{-1}) + (\Delta C_i - C_o).
\]

The payoffs from projects that existed before awarding the options are represented by the \(C_i\) cash flow. Hence, management receives a portion of these cash flows, even though they would have occurred even had management not been awarded the options.

Proof of Proposition 7

For simplicity assume a single share outstanding at the start. The available cash, \(C_0\), is sufficient to purchase a fraction, \(C_0/(C_0 + C_i r^i)\), of the shares. The stock price after repurchase will be \(C_i r^i/(1 - (C_0/(C_0 + C_i r^i))) = C_0 + C_i r^i\), the same price as before the repurchase. The option value is \((C_i/(1 - C_0/(C_0 + C_i r^i)) - (C_0 + C_i r^i))r^i\) if positive and zero otherwise. With rearrangement of this equation, we find that the option value is positive if \(C_i - C_i r^i > C_0 - C_0 r\). The left-hand side is positive and the right-hand side is negative. Thus, this statement is always true, so options will always have value if the funds are used to repurchase shares.

The Use of External Debt Financing

If the firm has an inadequate supply of cash on hand to undertake new projects, it can issue new debt or equity. Let us assume that the firm has no cash on hand and its value is comprised solely of the shareholders’ claim on the future cash, \(C_i r^i\). Now let it raise \(D_0\) in cash by issuing debt with face value of \(D_0\). The value of the firm with debt financing will then be \(V_0 = D_0 + C_i r^i\), and the current stock price will be \(S_0 = V_0 - D_0 = C_i r^i\), the same value prior to the new issue of debt. After taking into account the
NPV of the new project with debt financing, the new stock price becomes \( S_0' = \Delta C_1 r^{i} + C_1 r^{-1} - D_0 \).

The payoff of the option with debt financing is \( \text{Max}(0, C_1 + \Delta C_1 - r D_0 - C_1 r^{i}) \).

This expression is positive if \((\Delta C_1 - r D_0) + (C_1 - C_1 r^{i}) > 0\). The second term in parentheses is clearly positive. Positive NPV is defined as \( \Delta C_1 r^{i} > D_0 \), which clearly is equivalent to the first condition in parentheses. Thus, the option payoff is positive with positive NPV.

The cost of the options is \( \gamma (C_1 + \Delta C_1 - r D_0 - C_0 r^{i}) r^{i} \). Below we see that this cost can be higher than the NPV of the new project. First we note that:

\[
\gamma \left( C_1 + \Delta C_1 - r D_0 - C_0 r^{i} \right) r^{i} > \Delta C_1 r^{i} - D_0
\]

\[
\gamma \left( C_1 + \Delta C_1 - r D_0 - C_1 r^{i} \right) > \Delta C_1 - r D_0
\]

\[
\gamma > \frac{\Delta C_1 - r D_0}{C_1 + \Delta C_1 - r D_0 - C_1 r^{i}}.
\]

To show the cost is higher than the NPV, we need only show that the ratio on the right hand side is less than one.

\[
\Delta C_1 - r D_0 < C_1 + \Delta C_1 - r D_0 - C_0 r^{-1}
\]

\[
0 < C_1 - C_1 r^{-1}.
\]

With \( r \) positive, the cost of the options can be higher than the NPV of the new project.

We showed that the option payoff is positive if \((\Delta C_1 - r D_0) + (C_1 - C_1 r^{i}) > 0\). With negative-NPV the first term in parentheses is negative, but it can be outweighed by the second term in parentheses. Note also that managers clearly share in value created by projects in place before the options are awarded.

If there is no positive-NPV project and debt is issued to pay out a dividend, then it will be difficult for managers to obtain a positive option payoff. The option payoff is positive if

\[
C_1 - r D_0 > C_1 r^{-1}
\]

\[
C_1 - C_1 r^{-1} > r D_0.
\]

It will be difficult to meet this condition. The time value on the firm’s existing cash flow would have to exceed the principal and interest on the debt. When the strike price is adjusted by the dividend payout, it is easier for management to obtain a positive payoff, as shown below:

\[
C_1 - r D_0 > C_1 r^{-1} - D_0
\]

\[
C_1 - C_1 r^{-1} > r D_0 - D_0.
\]

This condition is more easily met than in the case of no strike adjustment.
If the manager decides to repurchase some shares by using debt financing, the result will be the same as the dividend payout case without adjustment. With $D_0$, the firm can purchase a fraction $D_0/C_1 r^{-1}$ of the shares. The stock price after repurchase will be \[ (C_1 - rD_0)/[1 - D_0/C_1 r^{-1}] = C_1 r^{-1}, \] the same price as before the repurchase. The option payoff is $(C_1 - rD_0 - S_0) = (C_1 - rD_0 - C_1 r^{-1})$ if positive and zero otherwise. The expression on the right-hand side is the same as the dividend case without adjustment of the strike price, and, therefore, would be unlikely to be positive.

**The Use of External Equity Financing**

Now let us consider the case that the new project is financed by issuing $\theta_0$ of new equity. The new equity will result in the issue of $\theta_0/S_0$ new shares making the total number of shares be $(\theta_0 + S_0)/S_0$. The stock price remains at $S_0 = C_1 r^{-1}$. A positive option payoff is the condition

\[
\frac{\Delta C_1 + C_1}{(\theta_0 + S_0)/S_0} > S_0.
\]

Multiplying both sides by $(\theta_0 + S_0)/S_0$ and substituting $C_1 r^{-1}$ for $S_0$ gives

\[
\Delta C_1 + C_1 \left(\frac{S_0 + \theta_0}{S_0}\right) S_0 \\
\Delta C_1 - \theta_0 + C_1 - C_1 r^{-1} > 0
\]

With positive NPV, $\Delta C_1 - \theta_0 > 0$. Thus, positive NPV is sufficient for the option payoff to be positive. It is also apparent from the above equation that the option payoff can be positive with negative NPV. For example, it is possible that $\Delta C_1 r^{-1} < \theta_0$ but $\Delta C_1 > \theta_0$, giving a positive option payoff. The cost of the options exceeds NPV if the following condition holds:

\[
\gamma \left(\frac{C_1 + \Delta C_1}{(\theta_0 + S_0)/S_0} - S_0\right) r^{-1} > \Delta C_1 r^{-1} - \theta_0
\]

\[
\gamma > \frac{\Delta C_1 - r\theta_0}{C_1 + \Delta C_1}/\left((\theta_0 + S_0)/S_0\right) - S_0
\]

Since $\gamma$ can be as large as one, we need only prove that the ratio on the right-hand side can be less than one. Define $\alpha = S_0/(\theta_0 + S_0)$. The proof is as follows.
It can be shown that $\alpha$ can exceed $r^{-1}$. This result occurs if $\theta_0/S_0$ is greater than the cost of capital, $r - 1$. The expression $\Delta C_1(1 - \alpha) - \theta_0 r$ can be negative, even with positive net present value. So the overall expression can be negative, thereby admitting a critical $\gamma$ of less than 1, which permits cost to exceed net present value. It is apparent that as in the other cases, part of the option’s payoff comes from the projects that the firm had in place before the options were awarded.

The case that the firm uses equity financing to pay out dividends or repurchase shares does not have economic meaning here. Therefore, we do not analyze these cases.

*Capital Investment is less than the Available Cash*

We assume that the firm invests $\rho C_0$ and has $(1 - \rho)C_0$ available for dividends, share repurchase, or financial investment. The incremental cash flow is $\Delta C_1$ and the capital investment is $\rho C_0$.

If the firm pays dividends, the cash flow at time 1 is $C_1 + \Delta C_1$. We assume the standard contract in which there is no dividend adjustment of the strike. Thus, the option payoff is determined by $Max(0, C_1 + \Delta C_1 - (C_0 + C_1 r^{-1}))$. This value is positive if $C_1 - C_1 r^{-1} + \Delta C_1 - C_0$. We have seen this expression before and clearly the option has value for any positive $NPV$ project. In addition, we could have $NPV < 0$ (such as $\Delta C_1 = C_0$) and the option will still have positive value. In general, $NPV < 0$ occurs as $\Delta C_1 < \rho C_0$. We can certainly have $\Delta C_1 \geq C_0$ if $\Delta C_1/C_0 \geq \rho$, which is sufficient but not necessary for managers to gain with negative $NPV$.

If the firm invests $(1 - \rho)C_0$ in financial investment, the option payoff is determined by $Max(0, C_1 + \Delta C_1 + (1 - \rho)C_0 r - (C_0 + C_1 r^{-1}))$. This value is positive if $C_1 - C_1 r^{-1} + \Delta C_1 + (1 - \rho)C_0 r - C_0 > 0$, which can obviously occur. If $NPV$ is negative, then let us rewrite this expression as $(C_1 - C_1 r^{-1}) + (\Delta C_1 - \rho C_0 r) + C_0(r - 1)$. The

\[\frac{\Delta C_1 - \theta_0 r}{(\theta_0 + S_0)/S_0 - S_0} < 1\]

\[\Delta C_1 - \theta_0 r < \frac{C_1 + \Delta C_1}{(\theta_0 + S_0)/S_0 - S_0}\]

\[\Delta C_1 - \theta_0 r < (C_1 + \Delta C_1)\alpha - C_1 r^{-1}\]

\[\Delta C_1(1 - \alpha) - \theta_0 r - C_1(\alpha - r^{-1}) < 0\]
second term in parentheses represents the negative \( NPV \) but the first and third terms are positive so clearly negative \( NPV \) does not rule out a gain for management.

If the firm invests \((1 - \rho)C_0\) in share repurchase, it buys back \(((1 - \rho)C_0)/(C_0 + C_1r^1)\) shares, leaving \(1 - (((1 - \rho)C_0)/(C_0 + C_1r^1))\) shares. The value of the firm at time 1 will be \(C_1 + \Delta C_1\) so the option payoff will be \(\text{Max}(0,(C_1 + \Delta C_1)/(C_1r^1 + \rho C_0)/(C_0 + C_1r^1)) - (C_0 + C_1r^1)\). This expression is positive with a positive \( NPV \), but it can also be positive with negative \( NPV \).
References


