Order imbalance period by period

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First version: June 2006
This version: March 2008

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Abstract

Using Poisson order arrivals, we introduce a simple procedure for testing the null hypothesis that the number of buyer-initiated trades equals the number of seller-initiated trades. A good (bad) news period contains informed trading, and, hence, is a period in which we reject the null hypothesis in favor of the alternative that the number of buyer-initiated (seller-initiated) trades is greater than the number of seller-initiated (buyer-initiated) trades. No-news periods contain only uninformed trading, and, hence, are periods for which we cannot reject the null hypothesis. We illustrate our approach using both simulated and transactions data.

JEL code: C12, G12

Key words: informed trading, trade arrivals

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1. Introduction

Using New York Stock Exchange tick data, Chordia, Roll and Subrahmanyam (2002) and Chordia and Subrahmanyam (2004) show that prices and liquidity are related to order imbalance, the difference between the number of buy orders and the number of sell orders. They report that order imbalance is significantly associated with daily changes in liquidity and contemporaneous market returns. Lee (1992) examines order imbalance around earnings announcements. Brown, Walsh, Yuen (1997) investigates the relation between order imbalance and return. Other studies, such as Blume, MacKinlay and Terker (1989) and Lauterbach and Ben-Zion (1993) who analyze the market crash of 1987, use alternate measures of order imbalance

We extend previous work by testing whether there is statistically significant order imbalance period by period. Using the procedure of Przyborowski and Wilenski (1940), we test whether buys equals sells for each period. Hence, we are able to determine, at the end of a trading period, whether a statistically significant order imbalance occurred. We assume that the arrival rates of buys and sells are governed by independent Poisson processes.[1] Our approach exploits the idea that trade count imbalances contain information about the arrival rate of informed trades and the number of balanced trades contains information on the arrival rate of uninformed trades. Hence, it is natural to associate periods with an abnormally high number of buyer-initiated trades (buys) with good news, an abnormally high number of seller-initiated trades (sells) with bad news, and a period for which we cannot reject that

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[1] Many authors including Back and Baruch (2004), Easley, Kiefer, O'Hara, and Paperman (1996) and Easley, Hvidkjaer, and O'Hara (2002) also assume Poisson trade arrival rates.

buys equals sells with no news. The information associated with this news can be market wide or firm specific.

On 17 January 2006, stock prices plunged 2.8% on the Tokyo Stock Exchange (TSE) and on the following day, the exchange closed early due to an influx of orders.[2] The market plunge was triggered by the event of government officials raiding the offices of internet firm Livedoor. We test the null hypothesis that buys equal sells for the days surrounding this event for each firm traded on the TSE. We show that the number of firms with statistically significant order imbalance is substantially greater on the event day and the following day than on other days. But even on these two days there were 5 and 9 firms, respectively, with significantly more buyer-initiated than seller-initiated trades. We believe that this application provides evidence of the usefulness of our approach.

In related research, a series of seminal papers [including Easley, Kiefer, O'Hara, and Paperman, 1996, (EKOP) and Easley, Hvidkjaer, and O'Hara, 2002, (EHO)] develop a model of and use a sophisticated optimization approach to estimate the probability of informed trading (PIN). These authors obtain a single estimate of PIN for a time series of daily buys and sells. In contrast, our approach is period by period and our test statistic can be estimated more simply without the use of optimization. Using simulated data, we show that our approach is related to PIN in that the number of rejections of the hypothesis of buys equals sells over a period of days provides an estimate of the probability of informed trading.[3]

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[2] We prefer to use TSE data rather than U.S. data to avoid difficulties in identifying buyer- and seller-initiated trades.

[3] The output of the EKOP model can, however, be used to derive confidence bounds on the number of buyer- and seller-initiated trades. The potential advantage of such a procedure is that (since estimates of the trading intensities are obtained from the model) the null hypothesis can be tested against a specified alternative. Also, the EKOP model can easily be

2. A statistical test of order imbalance

Our goal is to develop a period-by-period test of order imbalance. Let $\theta_i = (\varepsilon_i, \mu_i)$ denote the true values of the arrival rates of uninformed and informed trades, respectively, in the i^{th} trading period. Values of ε and μ are news state dependent so that $E(\theta|I_N) = \varepsilon$, $E(\theta|I_G) = \varepsilon + \mu$ and $E(_i|I_B) = \varepsilon + \mu$, where $I_{N,}$ $I_{G,}$ and I_B are periods with no news, good news, or bad news, respectively. We assume that ε and μ are governed by independent Poisson processes. We require that the hypothesis $\mu > 0$ is rejected at a level of significance, $\o$. In this way we are able to obtain an indication of the trading periods in which a market participant with Poisson trade arrival beliefs can test *ex post* whether informed trades have arrived. Specifically, we test the following hypothesis:

Hypothesis 1.

Null (H_0): The arrival rate of informed, seller-initiated trades is zero. The true parameters satisfy $\varepsilon + \mu = \varepsilon$ or $\mu = 0$.

Alternate (H_1): The arrival rate of informed, seller-initiated trades is positive. The true parameters satisfy $\varepsilon + \mu > \varepsilon$ or $\mu > 0$.

Information available concerning the state of news in the market is the count of buyer-initiated trades (buys), b_i, and seller-initiated trades (sells), s_i. Again, we assume that the arrival of b_i and s_i are according to independent Poisson processes. Estimates of these event sets are defined **at the end of each trading period**, but before the start of the next trading period. If we reject the null hypothesis of equality of buys and sells in favor of buys > sells (sells > buys), it is natural to label the period

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extended to allow for different arrival rates of uninformed buy and sell orders. We thank the referee for these insights.

as a good (bad) news period. Otherwise, the period contains no news. Hence, by using a simple rule, market participants can decide from the cumulative trades observed at the end of a trading period what the type of news state has occurred.

Przyborowski and Wilenski (1940) develop, present critical values for, and discuss the power of a test when x_1 and x_2 are two independent random variables distributed according to the Poisson law and seek to test that the expectations m_1 and m_2 are the same. These authors indicate that when n is greater than 80 then $(x_1 - x_2)/\sqrt{(x_1 + x_2)}$ may be regarded as a unit normal deviate and x_1 is normally distributed about $\frac{1}{2}n$ with a standard deviation of $\frac{1}{2}\sqrt{n}$.

Tests of Hypothesis 1 involve tests of the hypothesis $b_i = s_i$. To see this note that when in the empirical likelihood $b_i > s_i$, and when B and S are independent Poisson distributed random variables, the conditional distribution of B, given $B + S = k_i$, is given by Przyborowski & Wilenski (1940). The null and maintained assumptions in Hypothesis 1 are equivalent to a test of the hypothesis

$$H_0: p_i = \tfrac{1}{2}$$

$$\text{against} \quad H_1: p_i > \tfrac{1}{2}$$

where p_i is the probability parameter of a binomial distribution. Note that since we know a priori that the level of informed trading is zero or positive, this hypothesis is one sided. We calculate parameter estimates using the $\varnothing$-level of significance to test Hypothesis 1 at the end of each trading period. We term these $\varnothing$-level estimates.

3. Simulation

Using simulated data, we examine the efficacy of our model, and also compare it with the PIN model. PIN is defined as

$$PIN = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon}$$

where α is the probability of a news event, μ is the informed trader arrival rate, and ε is the uninformed trader arrival rate. An additional parameter, δ, the probability of a news event being bad news is also defined.

To obtain estimates of α, δ, μ, and ε, EKOP and EHO maximize the following likelihood function:

$$L(B,S|\theta) = (1-\alpha)e^{-\varepsilon}\frac{\varepsilon^B}{B!}e^{-\varepsilon}\frac{\varepsilon^S}{S!} + \alpha\delta e^{-\varepsilon}\frac{\varepsilon^B}{B!}e^{-(\mu+\varepsilon)}\frac{(\mu+\varepsilon)^S}{S!}$$
$$+\alpha(1-\delta)e^{-(\mu+\varepsilon)}\frac{(\mu+\varepsilon)^B}{B!}e^{-\varepsilon}\frac{\varepsilon^S}{S!}$$

where B and S represent the number of buys and the number of sells on a given day, respectively. The model assumes that days are independent, therefore the likelihood of observing the buys and sells over *i* days, $M = (B_i, S_i)_{i=1}^{I}$ is the product of likelihoods:

$$L(M|\theta) = \prod_{i=1}^{I} L(B_i, S_i|\theta)$$

In our simulation, we follow EHO and let the probability of an information event, α, be 0.4 and the probability of bad news given an information event, δ, be 0.5. The arrival rate of informed and uninformed trades is Poisson with a mean of 40 and 50 per day, respectively. We simulate 90 trading days each for 4,445 firms or 400,000 individual stock days. Table 1 presents statistics showing that the simulated values reflect the assigned value well.

For each day, we test the hypothesis that buys = sells and compare the results for that day with the actual presence of informed trading from the simulation. There are 2,020 Type I errors and 4,533 Type II errors. We classify 78,654 days as bad news days, 78,569 days as good news days, and 242,677 days as no news days. Hence, our test results indicate that the probability of a news day is (78,654 + 78,569)/400,000 = 39.3%, which is very close to the actual value of α of 0.4. Since there are both buys and sells, the probability that an individual trade will be informed is 39.4%/2 = 19.65%. EHO calculate the probability of a trade being informed (PIN) as PIN = $\alpha\mu/(\alpha\mu + 2\varepsilon)$ or in our case .4(50)/((.4(50) + 2(40)) = 0.2, which is very close to the value we achieve using our approach. We are able to apply our approach to all stocks together because in the simulated data the means for each stock is the same. But we need to use individual firm results to get an individual firm probability.

In Table 2 we present statistics for our 400,000 stock days classified by state. The means of the number of daily buys and sells for bad, good, and no news days are: 39.53, 89.98; 90.04, 39.55; 40.43, 40.41. Note that these values recover the arrival rates of liquidity traders and informed traders, which are 40 and 50, respectively, almost exactly.

4. Empirical example

The Associated Press reported that on Tuesday January 17, 2006, the Tokyo Stock Exchange (TSE) plunged about 2.8% due to a raid by Japanese officials on the internet company Livedoor. On the following day, the exchange closed early due to an influx of orders that threatened to exceed the exchange's capacity.[4] We wish to examine order imbalance around this event. But first, it may be useful to describe the operation of the TSE.

In 2003 there were 2,174 companies listed on the TSE (TSE Fact Book, 2003). The TSE operates two trading sessions Monday through Friday from 9:00 to 11:00

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[4] http://en.wikipedia.org/wiki/Tokyo_Stock_Exchange

and 12:30 to 15:00. A call auction, referred to as the *itayose* method, is used to open and close each trading session. At other times, the TSE operates as a continuous auction, referred to as the *zaraba* method. Both limit and market orders are permitted. Comerton-Forde and Rydge (2006) provide a description of the institutional arrangements on the TSE.

Our analysis is based on a complete record of transactions and associated best bid and best ask quotes for all the stocks that are traded on the TSE for January 17, 2006 and the twenty days before and twenty days after the event day. The trade data are obtained from the *Reuters International* data maintained by the *Securities Industry Research Centre of Asia-Pacific*. For each day we classify each trade as buyer-initiated (seller-initiated) if it occurs at the ask (bid) price. To be included, we require a firm to have trades in our sample every day and only days with at least 20 trades are included. We apply our tests on the total count number of buyer and seller-initiated trades for each day for each stock. Our results are reported in Table 3. For January 16, 2006, the day before the plunge, 427 firms had significantly more sells than buys, 37 firms had significantly more buys than sells and we cannot reject the hypothesis that buys = sells for the remaining 1,083 firms. But on the day of the plunge and the following day, we reject the null hypothesis in favor of sells > buys for 184 and 352 firms, respectively. And only a few firms have significantly more buys than sells. Then, order imbalance returns to normal levels. Hence, our approach provides a way of statistically validating whether buys = sells and testing whether there is a statistically significant order imbalance.

For the 242 trading days for the year ended February 28, 2006, we calculate DIFFERENCE = the absolute value of the number of days for which buys > sells minus the number of days for which sell > buys. The mean and standard deviation of

DIFFERENCE are 28.10 and 34.17 (n = 242), respectively. Noting that $2.57(28.10/(34.17/\sqrt{242})) = 5.64$, we use an upper confidence value of 28.1 + 5.64 = 33.7 to identify days with market wide order imbalance. Days with an absolute value of DIFFERENCE above this level have statistically significant market wide order imbalance. There are 64 such days in our 242 sample. At that rate we expect 11 in our 41 day sample period so that with nine statistically significant differences, our 41-day period is slightly below normal in the occurrence of market wide order imbalance, indicating a period of market turmoil.

Over our 41 day sample period for our 1,154 firms, the total of Table 3, columns 2-4, are 1318, 2198, and 43279, respectively. The ratio of the number of days for which buys>sells (sells > buys) is 0.03 (0.051) or 0.81 together, indicating a low occurrence of order imbalance. Since our estimates vary with the confidence level, our results may be appropriately labeled as *ø*-level estimates.

We also compute PIN for our 1,154 firms for this 41 day period based on EKOP. As shown in Table 4, the mean PIN is 0.1299, which is substantially above the probability of informed trading using our measure. There is very little difference between the arrival rate of informed (μ) and uninformed trades (ε). Almost 31% of the days are identified as having news, which is greater than the 9/41 = 22% we identify. We leave a comprehensive comparison of our approach and PIN for future research.

6. Conclusion

For any period, we present an approach to test ex post whether the number of buys equals the number of sells when the arrival rates are independent and have a Poisson distribution. If we reject the null hypothesis of equality in favor of buys > sells (sells > buys), we conclude that these have been good (bad) news; otherwise, there is no news. Using simulated data, we show that our classification of days as having bad, good, or no news has a small number of type I and type II errors and recovers the simulated informed and uninformed arrival rates. Using our classification of days by news state, we calculate a measure of the probability of informed trading of 19.3%, which compares favorably with the true value of 20% known from the simulated parameters. We also illustrate out approach using actual data. On January 17, 2006, the Japanese government raided the internet firm Livedoor, causing a plunge in stock prices and an early closing of the exchange on the following day. Using a count of daily buys and sells for the Tokyo Stock Exchange, we show that many more firms have a statistically significant level of order imbalance on the day of the raid and on the following day than on other days.

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Table 1. Simulation statistics. We simulate 90 trading days for 4,445 firms. Our simulated parameter values $\alpha = 0.4$, $\delta = 0.5$, $\mu = 50$ per day, and $\varepsilon = 40$ per day. The arrival rates are distributed Poisson. Statistics for the simulated data are presented in columns 2-5. Note that the simulated values conform well to the assigned values.

Parameters	**Mean**	**Std Dev**	**Minimum**	**Maximum**
ε	40.00	0.530	38.186	41.858
μ	50.03	1.646	43.980	56.581
α	0.399	0.051	0.2223	0.6005
δ	0.501	0.084	0.2058	0.8571
PIN	0.199	0.021	0.1246	0.2732

Table 2. Simulation results by state. We present statistics for the 400,000 simulated stock days by state.

State:	**Bad**		**Good**		**No news**	
	buys	**sells**	**buys**	**sells**	**buys**	**sells**
N	78,754	78,754	78,569	78,569	242,677	242,677
MEAN	39.53	89.98	90.04	39.55	40.43	40.41
STD	6.24	9.96	10.03	6.22	7.13	7.11
MIN	15	37	37	13	13	15
MAX	68	130	133	67	98	96

Table 3. Results of test of buys = sells. On January 17, 2006 (t=0) stock prices plunged 2.8% on the Tokyo Stock Exchange and on the following day the exchange closed early due to an influx of orders. For days t-20 through t+20 relative to January 17, we test the null hypothesis that buys=sells, against the alternatives that buys > sells or sell > buys and indicate the number of firms for which each of these three outcomes applies. The number of firms can vary slightly each day because we require that the sum of buys plus sells equal 30 or more. Column 6 reports the absolute value of the difference between columns 2 and 3. We calculate the mean and standard deviation of DIFFERENCE for the 242 trading days ended February 28, 2006 and use these to calculate a confidence interval.. * indicates that the daily value of DIFFERENCE is statistically different from 0 at the 0.01 level.

Day	Buys>sells	Sells>buys	Sells=buys	Total	DIFFERENCE
t-20	6	103	1,043	1,152	97*
t-19	35	51	1,057	1,143	16
t-18	20	60	1,067	1,147	40
t-17	21	71	1,058	1,150	50
t-16	60	32	1,054	1,146	28
t-15	31	45	1,072	1,148	14
t-14	27	27	1,088	1,142	0
t-13	30	21	1,092	1,143	9
t-12	17	34	1,087	1,138	17
t-11	57	9	1,057	1,123	48
t-10	36	39	1,069	1,144	3
t-9	27	48	1,027	1,102	21
t-8	32	24	1,028	1,084	8
t-7	42	15	1,092	1,149	27
t-6	39	23	1,083	1,145	16
t-5	27	38	1,084	1,149	11
t-4	36	14	1,098	1,148	22
t-3	61	24	1,065	1,150	37
t-2	41	25	1,078	1,144	16
t-1	37	27	1,083	1,147	10
0	5	184	965	1,154	179*
t+1	9	352	793	1,154	343*
t+2	123	12	1,017	1,152	111*
t+3	23	76	1,053	1,152	53
t+4	18	93	1,039	1,150	75*
t+5	67	11	1,064	1,142	56*
t+6	34	34	1,075	1,143	0
t+7	53	21	1,064	1,138	32
t+8	36	20	1,090	1,146	16
t+9	12	37	1,099	1,148	25
t+10	23	30	1,082	1,135	7
t+11	19	41	1,072	1,132	22
t+12	17	34	1,088	1,139	17
t+13	32	24	1,072	1,128	8
t+14	36	16	1,081	1,133	20
t+15	30	29	1,070	1,129	1
t+16	9	75	1,053	1,137	66*
t+17	14	65	1,064	1,143	51
t+18	8	92	1,047	1,147	84*
t+19	14	190	944	1,148	176*
t+20	54	32	1,065	1,151	22

Table 4. PIN statistics. We estimate the parameters of the PIN model for the 41 day period centered on 17 January 2006.

Parameters	Mean	Std Dev	Minimum	Maximum
ε	97.60	90.47	3.85	967.42
μ	101.47	82.69	0	1309.91
α	0.3063	0.2311	0	1.0000
δ	0.5207	0.2409	0	1.0000
PIN	0.1299	0.0677	0	0.5890