Analysis Determinants of VPIN, HFTs' Order Flow Toxicity and Impact on Stock Price Variance

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Draft: September 09, 2013

Abstract

This study examines HFTs' order flow toxicity to both HFT and non-HFT liquidity suppliers, and HFTs' impact on stock price variance. Order flow toxicity is measured with VPIN metric. Determinants of order flow toxicity, relation between volatility and order flow toxicity, and application of FVPIN contract as a protection against order flow toxicity are also examined. An 'actual variance' measure which eliminates impact of spreads on the variance is developed. Results show that HFTs exert order flow toxicity to non-HFT liquidity suppliers. While trade intensity is negatively related to VPIN, return volatility is positively related to VPIN. VPIN has predictive power for future volatility in equity markets, even after controlling for trade intensity. FVPIN contract is a useful hedge tool against toxicity. The developed actual volatility measure shows that due to impact of spreads observed variance seems to be higher than actual one.

1. Introduction

High frequency trading is a subset of algorithmic trading that aims to profit from trading at very high speed and holding inventories for only seconds or milliseconds (Brogaard, 2010). The 26 high frequency trading firms, identified in the NASDAQ HFT dataset, which includes 120 stocks, participate in 74% of all trades which executes on NASDAQ and make around $3 billion annually (Brogaard). The upper boundary for estimated profit of aggressive high frequency traders (HFTs) on the US market is around $26 billion (Kearns et al., 2010). Cartea and Penalva (2011), Jarrow and Protter (2011) and Biais, Foucault, and Moinas (2011) develop theoretical models to understand the roles of HFTs. This theoretical work implies that HFTs may be harmful or beneficial for market quality under certain conditions. Empirical studies (Brogaard; Zhang, 2010; Kearns, Kulesza, and Nevmyvaka, 2010; Menkveld, 2011; Kirilenko, Kyle, Samadi, and Tuzun, 2011; and Brogaard, Hendershott, and Riordan, 2012) look at HFTs from different perspectives and find that HFTs appear to be mostly beneficial for markets.

This paper examines two issues related to HFTs, the order flow toxicity in HFT trades and the impact of HFTs on stock price variances. We also examine the determinants of order flow toxicity, the forecasting power of the VPIN metric for return volatility, and test the ability of FVPIN future contracts in protecting against order flow toxicity. While examining the variance impact of HFTs, we also utilize a volume based approach in calculating variance and develop an 'actual variance' measure that eliminates spreads' impact on the observed variance.

Empirical studies of HFTs mainly focus on the impact on price discovery, liquidity, spreads, and stock price volatility. Our study focuses on HFTs' impact on order flow toxicity. Easley et al. (2012-a) develop a new methodology – volume-synchronized probability of informed trading, the VPIN measure – to estimate order flow toxicity based on volume imbalances and trade intensity. This measure depends on volume time rather than clock time. Easley et al. show that the VPIN is an applicable measure for short term, toxicity induced volatility. By applying the VPIN approach, our study aims to determine the impact HFTs have on order flow toxicity and losses to liquidity providers. In extreme cases, high loses caused by HFTs may force liquidity providers out of the market, hence, the findings of this study can be used to

suggest microstructure alterations to maintain market stability. According to Easley, Prado, and O'Hara (2012-a) order arrivals contain information about the price movements and a volume based approach is more relevant to extract information than the clock time approach. Accordingly, applying the VPIN approach is more reliable to study the relation between liquidity suppliers and HFTs than methods that apply a clock time approach.

By examining the average VPIN of HFTs' trades we find that, regardless of trader type (HFT or non-HFT), the lowest toxicity occurs in trades of high volume stocks. HFT initiated trades have higher toxicity than non-HFT initiated trades in the overall sample and in all volume classifications, except the high volume sample. Trades in which both the liquidity demander and liquidity supplier are high frequency trading firms have the highest toxicity in all samples except high volume stocks. We find that trade toxicity is twice as high for transactions where both the liquidity supplier and demander are HFT firms than when neither side of the transaction is an HFT firm. The toxicity problem is more severe in low volume stocks than medium volume and high volume stocks. Based on our findings, which are consistent with Cartea and Panelva (2011), Biasis, Foucault and Moinas (2011), Jarrow and Protter(2011), and Brogaard, Hendershott and Rioardan (2012), we conclude that HFTs may cause losses to other liquidity providers.

Our study also examines the determinants of order flow toxicity. According to studies on VPIN metric (Easley et al. 2012-a) and factors that affect liquidity suppliers willingness (Griffiths et al., 2000) we expect trade intensity and risk to be important determinants of VPIN. We find that the number of trades, trade size per transaction, and volatility of the stock are main determinants of order flow toxicity. While trade intensity variables are negatively related to toxicity this relation is not linear. Also risk is positively related to toxicity.

There is an ongoing debate about the relation between volatility and the VPIN metric (Easley et al. 2012; Anderson and Bondarenko, 2013). While Easley et al. find VPIN and absolute returns are correlated, Anderson and Bondarenko find that VPIN has no predictive power for future volatility. Both of these studies test E-mini S&P 500 futures contracts data. We examine the relation between volatility

and VPIN metric on equity market. By using two different volatility measures we show that VPIN in volume bucket $\tau - 1$, is positively related to volatility even after controlling for trade intensity variables.

Easley, Prado, and O'Hara (2011) develop a futures contract (FVPIN) that is valued as [-ln(VPIN)] and is argued to hedge against the order flow toxicity. In our study, we test if FVPIN contracts can protect traders against flow toxicity by calculating the returns of FVPIN contracts over 120 stocks in 2009. Our findings show that the FVPIN futures contract can provide positive returns in the overall sample and all volume deciles, however, for a given level of return high volume sample provides the lowest risk. Overall, we conclude that FVPIN may be a hedging tool against the toxicity losses for liquidity suppliers.

The HFTs' impact on stock price variances is another issue we examine. While theoretical work predicts that HFTs' increase price volatility (Cartea and Penalva 2011; and Jarrow and Protter, 2011), the empirical results for HFTs impact on volatility is mixed. Brogaard (2010) finds that HFTs may reduce price volatility. On the other hand, Kirilenko, Kyle, Samadi, and Tuzun (2011) find that HFTs lead to an increase in volatility during the flash crash. Similar to Kirilenko et al., Zhang (2010) finds that HFTs may increase stock price volatility. In this study, we approach the HFTs—stock price volatility relation from a different perspective.

By building on Easley, Prado, and O'Hara's (2012-a) argument that, in general, a volume clock is more relevant than a time clock in a high frequency world for future price movements, we apply a volume based stock price variance calculation method, rather than the classical time clock approach, to examine HFTs impact on stock price volatility. Corwin and Schultz (2012) argue that a variance measure free from bid ask bounce may be useful for financial research. To this end, following Parkinson (1980) and Corwin and Schultz (2012), we develop a variance measure that takes into account and eliminates the impact of spread on observed variance.

Our results show that, when HFTs demand liquidity from non-HFTs they increase observed variance, which is consistent with theoretical predictions of Cartea and Penalva (2011) and Jarrow and Protter (2011) and the empirical findings of Zhang (2010). We find that when HFTs provide liquidity

they decrease variance, implying HFTs may reduce stock price volatility. This finding is consistent with Brogaard (2010).

The actual variance measure, which we develop in our paper, yields noteworthy results. First, when the impact of spread is removed, actual variance is always lower than observed variance. Second, while observed variance shows a significant, but small, difference between HFT and non-HFT trades, the actual variance model shows that there is no difference between the samples. These findings support Corwin and Schultz (2012) and O'Hara (2006) arguments that removing the impact of spreads from variance may be beneficial for financial research.

2. Hypotheses development and related literature

2.1 High frequency traders and order flow toxicity

Cartea and Penalva (2011), Jarrow and Protter (2011), and Biais, Foucault, and Moinas (2011) theoretically study the role of HFTs in financial markets. With a three-agent model (HFT, market maker, and liquidity trader), Cartea and Penalva propose that HFTs cause losses to both liquidity traders and market makers, increase price volatility and volume, but do not improve liquidity. Since market makers losses are compensated with higher liquidity discounts, HFTs' net impact on market maker profit is zero.

Similar to Cartea and Penalva's (2011) predictions, Jarrow and Protter (2011) show, with a theoretical model that assumes a frictionless and competitive market (no bid/ask spread, and perfectly liquid markets), that HFTs may have a dysfunctional role in financial markets. According to their model, HTFs, due to their high speed, can react to a signal (i.e. price change) much faster than ordinary investors and thus all HFTs react like a large trader with the same trade. So, HTFs both increase market volatility and create their own profit opportunities (price momentum) at the expense of ordinary traders.

Cartea and Penalva (2011) and Jarrow and Protter (2011) agree that HFTs generate losses to other traders, however Biais, Foucault, and Moinas (2011) find that increases in the level of high frequency trading, until a threshold, may increase the probability that investors will find a trading counterparty and thereby increase trading volume and profits. On the other hand, high levels of high

frequency trading can impose adverse selection costs on slow traders and reduce volume and profits and cause slow traders to drop out of the market.

Based on these theoretical works, we expect HFTs to increase adverse selection in trading and decrease profits or even cause loses to other traders. When order flow adversely affects liquidity providers and causes losses to them, it is called order flow toxicity (Easley, Prado, and O'Hara; 2012-a). As a result, we expect higher order flow toxicity in HFTs' trades than other trades. We formally test the following hypothesis.

Hypothesis 1: HFTs exert higher order flow toxicity on non-HFT liquidity suppliers than they do on HFT

liquidity suppliers.

Examining the impact of HFTs on order flow toxicity is important because order flow toxicity may affect market liquidity. Easley, Prado, and O'Hara, (2012-a) reason that high toxicity, which is measured by VPIN, will increase losses to liquidity providers; hence liquidity providers may drop out of the market and by extension, decrease liquidity. HFTs' impact on market liquidity is examined by several empirical studies namely; Hendershott, Jones, and Menkveld (2011), Hendershott and Riordan (2011), and Brogaard, Hendershott, and Riordan (2012).

Hendershott, Jones, and Menkveld (2011) study the impact of algorithmic trading (AT) on liquidity for a sample of 923 NYSE stocks over the five years from 2001 through 2005 and find that AT improves liquidity. Similarly, Hendershott and Riordan (2011) study the 30 largest DAX stocks and the role of algorithmic traders in market quality and find that AT consumes liquidity when spreads are wide (liquidity is expensive) and provides liquidity when spreads are narrow (liquidity is cheap). On the other hand, Brogaard, Hendershott, and Riordan (2012), who examine 120 randomly selected NASDAQ stocks from 2008 to 2009, find that HFTs' non-marketable orders may cause other liquidity holders to withdraw from the market.

We determine what impact (if any) HFTs have on market liquidity through order flow toxicity. Though current studies examine the direct impact of HFTs on market liquidity and find that HFTs mostly increase liquidity, our approach is different than current empirical HFT studies (Brogaard et al., 2012 ;

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Hendershott et al., 2011, and Hendershott and Riordan, 2011) in that we examine the indirect impact (if any) HFTs have on liquidity through order flow toxicity. In short, if toxicity of HFTs' order flow is higher than that of normal investors (if hypothesis 1 is supported), then relying on Easley, Prado, and O'Hara, (2012-a) reasoning, we can conclude that HFTs may harm market liquidity indirectly as they may cause other investors to drop out the market.

2.2 Determinants of the VPIN metric

Another contribution of our study is that we examine the determinants of the VPIN measure. According to Easley et al. (2012 –a), VPIN is a measure of order flow toxicity -order flow is toxic when it causes losses to liquidity suppliers- and based on volume imbalances and trade intensity. Accordingly in our examination of the determinants of VPIN, we focus on trade characteristics such as size and number of trades, as well as factors that affect liquidity providers' willingness to provide liquidity.

Griffiths et al. (2000) cite that trader's willingness to supply liquidity is a decreasing function of stock price variability. Accordingly, we expect variance in stock returns to affect order flow toxicity measure "VPIN". We calculate our proxy for price variability by dividing each volume bucket into ten equal sub-volume buckets. Using the first and the last price in each sub-bucket we calculate the percentage change in prices. We then calculate standard deviations by using the percentage change in prices. We call this measure as "Risk" variable in the multivariate analysis.

Depending on Easley et al. (2012–a) VPIN definition, we expect trade intensity to be an important determinant. We measure trade intensity with two different measures, average trade size and number of trades in each volume bucket. The average trade size is the number of shares per trade in each volume bucket. Easley et al. also study the distribution of VPIN conditional on absolute returns. They find that high absolute returns are rarely followed by small VPIN. Thus, we include absolute return in each volume bucket in our analysis as a determinant of VPIN. Specifically absolute return is defined as

$|\frac{P_\tau}{P_{\tau-1}} - 1|$, where P_i is the average price in volume bucket i.

We formally test the following hypothesis regarding the determinants of VPIN:

Hypothesis 2: Trade intensity and return volatility are significant determinants of the order flow toxicity measure VPIN.

2.3 Relation between risk, absolute return and VPIN Metric

Easley et al. (2012-a) find that there is a linkage between toxicity and future price movements. Specifically, the authors determine that VPINs are positively correlated with future price volatility for E-mini S&P 500 futures data. Accordingly, they conclude that the VPIN has a significant predictive power over toxicity induced volatility. On the other hand, by studying E-mini S&P 500 futures contracts, Anderson and Bondarenko (2013) conclude that once trading intensity and volatility are controlled, VPIN metric has no incremental forecasting power for future volatility. The volatility in Easley et al. paper is defined as percentage change in prices between two subsequent volume buckets (absolute return), $|\frac{P_{\tau}}{P_{\tau-1}}-1|$, where P_i is the average price in volume bucket i. In Anderson and Bondarenko, volatility is proxied with average absolute one minute return (AAR) where forecast horizon is defined as five minutes and one day.

We test the relation between return volatility and VPIN in equity markets. Our VPIN metric does not suffer from any order classification errors because our data specify the trade initiator. Following both Easley et al. (2012-a) and Anderson and Bondarenko (2013), we measure volatility with two different measures, absolute return and average return volatility. Absolute return is definition is given above. For average return volatility, since VPIN measure is mainly depend on volume time not clock time, we divide each volume bucket into ten equal sub volume intervals, and calculate return volatility by using these ten sub volume intervals. Formally we test the following hypothesis without arguing the direction of the relation.

Hypothesis 3: The order flow toxicity in volume bucket τ will be related to volatility in volume bucket $\tau+1$.

2.4 Protection against order flow toxicity, FVPIN contracts

We further provide empirical evidence on the application of a futures contract that may protect investors against order flow toxicity. Easley, Prado, and O'Hara, (2011) suggest a futures contract (FVPIN), which is valued as [-ln (VPIN)], can be used as a hedging instrument against order flow toxicity. We formally test the following hypothesis regarding the FVPIN contract.

Hypothesis 4: FVPIN future contracts provide positive returns to investors.

Easley, Prado, and O'Hara (2012-b) argue that HFTs are not temporary traders in the market place, hence, non-HFTs must adapt to the new trading environment. FVPIN futures contracts may be one way that non-HFTs can adapt. If these contracts can protect investors against order toxicity, they can help non-HFTs deal with possible HFT induced order flow toxicity.

2.5 High frequency traders' impacts on volatility

Another prediction of theoretical studies (Cartea and Penalva, 2011, and Jarrow and Protter, 2011) is that HFTs increase volume and volatility in markets. We contribute to this stream of literature in two ways: First unlike previous literature, we apply a volume based variance method when examining the impact of HFTs. Easley, Prado, and O'Hara (2012-a) find that volume time, which may be more relevant to high frequency world, is much closer to the normal distribution, has less heteroscedasticity and serial correlation than clock time.

Second, by building on results of Parkinson (1980) and Corwin and Schultz (2012), we develop a variance measure that eliminates the impact of the spread on observed variance. We define the variance calculated with this measure as actual variance. O'Hara (2006) argues that, since asset prices emerge in financial markets, the costs of liquidity (i.e. spreads) may matter for the prices. Since spread may affect prices, spreads may also affect variance. This reasoning is supported by Corwin and Schultz, who argue that a variance measure that is free from bid-ask spread bounce may be beneficial for financial research.

Kirilenko et al. (2011) examine the trading patterns of HFTs on a single day (May, 6, 2010) and do not consider the direct impact of HFTs on stock price volatility for a longer period. Their study focuses on overall market volatility, which is caused by HFTs' trades on May 6, 2010. The main differences

between our study of HFTs' impact on price volatility and that of the previous studies is that, unlike Kirilenko et al., our focus is on stock price volatility impact of HFTs in a more general setting (over an entire year, 2009, and over one week in 2010, not for a single event day). Brogaard (2010) calculates realized volatility in one minute intervals with and without HFT initiated trades and then compares realized volatility with volatility calculated without HFT initiated trades. In our study, we examine the trades in six different samples without making any assumptions about the existence of HFTs in the market. As a result our sample is more comprehensive and less likely to violate any microstructure properties. Another unique feature of our study is that we separate realized volatility and actual volatility in a volume   based setting. Zhang (2010) controls for factors that have an impact on stock price volatility, tests the impact of HFT on the volatility, and finds that HFTs increase stock volatility. In his data set HFT trading is not directly observable; rather he uses an estimated HFT activity measure. Unlike his study, we directly observe the initiator of the trade (HFTs or non-HFT).

By using results of Parkinson (1980) and assumptions of Corwin and Schultz (2012), we show that

$$\sigma^2_{observed,t} = k_1\sigma^2_{actual,t} + 2k_2\sigma^2_{actual,t} \ln\left(\frac{2+S}{2-S}\right) + 2\left[\ln\left(\frac{2+S}{2-S}\right)\right]^2 \qquad (eq.1)$$

Where $\sigma^2_{observed,t}$ is the observed variance of the stock prices in time t, $\sigma^2_{actual,t}$ is the actual variance of stock prices in time t, S is spread, $k_1 = 4\ln(2)$ and $k_2 = \sqrt{\frac{8}{\pi}}$. By solving equation 1 for actual variance and using observed spread data, we examine the impact of HFTs and non-HFTs on actual variance of stock prices. Our study separates the impacts of HFTs and non-HFT on actual stock variances and observed stock variances. By using our *actual variance measure* we test the following hypotheses:

Hypothesis 5: HFTs will increase observed and actual stock price variance.

We justify our study of the impact of HFTs on stock price volatility by the following: First, increases in stock volatility can increase the expected riskiness of the firm and, as a result, increase the

cost of capital (Froot, Perold, and Stein, 1992). As stock prices become noisier signals of firm value, stock price based compensations will become more costly (Baiman and Verrecchia, 1995). High stock price volatility (sudden and large stock price drops) can increase the likelihood of shareholder lawsuits (Francis, Philbrick, and Schipper, 1994).

3. Methodology

In this section we explain the methodologies we follow to calculate VPIN and volume bucket based observed variance and actual variance measures.

3.1 Order flow toxicity

We calculate volume-synchronized probability of informed trading, the VPIN toxicity measure, following Easley, Prado, and O'Hara (2012-a). This study defines VPIN as:

$$VPIN = \frac{\sum_{\tau=1}^{n} \left| V_\tau^S - V_\tau^B \right|}{n*V}$$

In the VPIN calculation Easley, Prado, and O'Hara first choose a fixed volume bucket size

($V = 1/50 * Average\ daily\ volume$) and set the number of buckets to $n = 50$. Then they calculate sale

volume (V_τ^S) and buy volume (V_τ^B) order imbalances for a given volume bucket (τ), and sum the order

imbalances over the number of buckets. VPIN is calculated by dividing the sum of order imbalances over the number of buckets by the number of buckets multiplied by bucket volume. In Easley, Prado, and O'Hara each volume bucket is divided into one minute time intervals to find buy volume and sale volume. Since we know if the trade is a buy or sale, we use the actual buy and sale volume instead of estimated ones. We expect actual data to increase the accuracy of the VPIN measure. We choose the bucket size,

$V = 1/50 * Average\ daily\ volume$ and the number of buckets, $n = 50$, as in Easley, Prado. The authors show that the choices of bucket size and number of buckets are robust to alternative specifications.

3.2 Impact of HFTs on actual stock variances

This section describes the methodology that we develop and use to study the impact of HFTs on stock price variances. Section 3.2.1 explains differences between volume interval and time interval approaches to variance calculation. Section 3.2.2 explains the two different methods we use to calculate volume interval variance, namely; observed variance and actual variance.

3.2.1 Time interval variance versus volume interval variance

Easley, Prado, and O'Hara (2012-a) reason that trade time is better captured by volume than clock time in a high frequency world, and that the order arrival process is informative about subsequent price movements. Measuring variance with a volume interval approach may be more beneficial than the classical time interval approach in a sample with high frequency traders. To better understand this reasoning, a simple quantitative explanation may be helpful.

We consider two cases. In first case we assume order flow will be fast and so volume buckets will be filled quickly, and in the second case we assume order flow is slow and volume buckets will be filled slowly. Consider a time interval $t_0 = 0, \& \, t_1 = 1$ in the first case, with a normal time interval approach. Basically, the variance will be a ratio of the highest price (P_h^t) to the lowest price (P_L^t). When order flow is fast and volume buckets are filling quickly, a fixed time interval will have more than one volume bucket (i.e. two volume buckets). Hence, the volume is high enough so that the interval is divided into multiple volume buckets (i.e. $\tau_0 \, to \, \tau_1 \, \& \, \tau_1 \, to \, \tau_2$). With the volume interval approach, we use the two highest prices, in this instance, and the two lowest prices to calculate the variance, implying that volume based measure becomes more sensitive than the time interval approach when order arrival is high because the volume interval approach considers more than one set of high and low price pairs, while the time interval approach only considers one set of high price and low price pairs, regardless of order arrival speed.

In case two, we assume that order arrival is slow so that it takes more time to fill the volume bucket, hence time bar interval will be a subset of the volume bucket. To calculate variance in case two

using a volume interval we will select the highest price (P_h^τ) and the lowest price (P_L^τ) in the volume bucket. Note that $P_h^\tau = \max(P_i \in \tau)$ and $P_L^\tau = \min(P_i \in \tau)$, the time interval is a subset of volume interval, $P_h^\tau \geq P_h^t$ and $P_L^\tau \leq P_L^t$. Hence, the volume interval will estimate a variance that is larger than or equal to time interval variance i.e. $(P_h^\tau / P_L^\tau) \geq (P_h^t / P_L^t)$.

Overall, the volume time approach is more sensitive when order arrival rates are high and does not underestimate the variance when order arrival rates are slow. A volume time (or interval) approach of calculating variance seems to be more beneficial in a high frequency world as suggested by Easley, Prado, and O'Hara (2012). We calculate volume interval based variance in two ways: an actual variance and an observed variance, which is explained in the following section.

3.2.2 Observed Variance versus Actual Variance

Building on Corwin and Schultz (2012), we assume that, in each volume bucket, the highest price trades are buyer initiated trades. Hence, the highest observed price (H_o^v) in volume bucket v is increased by half of the spread ($\frac{s}{2}$), the lowest price trades are assumed to be seller initiated trades and so the lowest observed price (L_o^v) in volume bucket v is decreased by half of the spread ($\frac{s}{2}$). Formally, the highest observed price is defined as $H_o^v = H_A^v * (1 + \frac{s}{2})$ where H_A^v is the highest price in the volume bucket v and the lowest observed price is defined as $L_o^v = L_A^v * (1 - \frac{s}{2})$ where L_A^v is the lowest price in the volume bucket v. Following Parkinson (1980) and Corwin and Schultz (2012), we define the variance as the natural log of the ratio of highest price to lowest price in each volume bucket.

$$[\ln(\frac{H_o^v}{L_o^v})]^2 = [\ln(\frac{H_A^v + H_A^v * s/2}{L_A^v - L_A^v * s/2})]^2 \qquad \text{(eq.2)}$$

Expanding eq.2 yields eq.3.

$$[\ln(\frac{H_o^v}{L_o^v})]^2 = [\ln(\frac{H_A^v}{L_A^v})]^2 + 2*\ln(\frac{H_A^v}{L_A^v})*\ln(\frac{2+s}{2-s}) + \ln(\frac{2+s}{2-s})^2 \qquad \text{(eq.3)}$$

The expected value of eq.3 yields eq.4.

$$E([\ln(\frac{H_o^v}{L_o^v})]^2) = E([\ln(\frac{H_A^v}{L_A^v})]^2) + 2*E(\ln(\frac{H_A^v}{L_A^v})*\ln(\frac{2+s}{2-s})) + E(\ln(\frac{2+s}{2-s})^2) \text{(eq.4)}$$

Since expected value is a linear operator and spread is an observed value, eq.4 can be rewritten as eq.5.

$$E([\ln(\frac{H_o^v}{L_o^v})]^2) = E([\ln(\frac{H_A^v}{L_A^v})]^2) + 2*E(\ln(\frac{H_A^v}{L_A^v}))*\ln(\frac{2+s}{2-s}) + \ln(\frac{2+s}{2-s})^2 \quad \text{(eq.5)}$$

Parkinson (1980) shows that (i) and (ii) hold.

i) $E(\ln(\frac{H}{L})) = \sqrt{\dfrac{8*\sigma_{HL}^2}{\pi}}$

ii) $E([\ln(\frac{H}{L})]^2) = (4*\ln 2)*\sigma_{HL}^2$ where $\sigma_{HL}^2 = \dfrac{0.361}{n}\sum_{i=1}^{n}l_i^2$ and $l_i = \ln(\frac{H_i}{L_i}), i = 1,2\ldots n$

Applying the findings of Parkinson (1980) to eq.5 yields eq.6.

$$(4*\ln 2*\sigma_{H_oL_o}^2) = (4*\ln 2*\sigma_{H_AL_A}^2) + 2*E(\ln(\frac{H_a^v}{L_a^v}))*\ln(\frac{2+s}{2-s}) + \ln(\frac{2+s}{2-s})^2 \qquad \text{(eq.6)}$$

Define $c = \ln(\frac{2+s}{2-s})$, $a = 4*\ln 2$ and $k = 2*\sqrt{\dfrac{8}{\pi}}$

$$a*\sigma_{H_oL_o}^2 = a*\sigma_{H_AL_A}^2 + k*c*\sigma_{H_AL_A} + c^2 \qquad \text{(eq.7)}$$

Rearranging the terms in (eq.7) yields

$$a*\sigma_{H_AL_A}^2 + k*c*\sigma_{H_AL_A} + (c^2 - a*\sigma_{H_oL_o}^2) = 0, \qquad \text{(eq.8)}$$

which can be solved for the actual variance of ($\sigma_{H_AL_A}^2$) given observed variance ($\sigma_{H_oL_o}^2$).

The root of (eq.8) is:

$$(\sigma_{H_A L_A})_{1,2} = \frac{-k*c \mp \sqrt{(k*c)^2 - 4*a*(c^2 - a*\sigma_{H_o L_o}^2)}}{2*a}$$ (eq.9)

The positive root of (eq.8) gives the actual variance of the stock price that is calculated by using the spread and observed variance in each volume bucket. We consider each volume bucket as an interval and calculate a volume bucket effective spread, S_v which equals two times the absolute value of the trade price minus the midpoint of the highest bid in volume bucket and the lowest ask in volume bucket. Formally:

$$S_v = 2*|p - (bid_{max} + ask_{min})/2|$$ (eq.10)

4. Sample and data

The dataset for this study comes from the NASDAQ high frequency trading database, which has trades of 120 select NASDAQ stocks. We use data from January 2009 through December 2009 and quotes from February 22, 2010 to Februarys 26, 2010. These are the only days of quote data in NASDAQ HFT database. These stocks vary in terms of volume and market capitalization. The trade data has a millisecond timestamp and an indicator of the initiator of the trade HFT trades are labeled with an H and non-HFT trades are labeled with an N. The dataset contains the data of 26 trading firms, which are identified as high frequency traders by NASDAQ. We also use the Center for Research in Security Prices (CRSP) data base to obtain volume, and number of shares outstanding of the stocks during 2009.

5. Descriptive Statistics

We have four types of trades in the HFT data. A high frequency trading firm on both sides of transactions is (HH) sample. A high frequency trading firm initiating a trade and a non HFT firm providing liquidity is (HN) sample. A non HFT trading firm initiating a trade and a HFT trading firm providing liquidity (NH), and no HFT trading firm on either side of the transaction (NN). While we look

at each of these four types of trades, we also look at trades in which HFT trading firms are the initiators of trades (HH_HN), and trades in which the non HFT are the trade initiators (NN_NH).

Table 1 reports the descriptive statistics of the VPIN measure and other variables that will be used in our analysis. Consistent with Easley et al. (2012-a; 2011) VPIN measure is between zero and one. Another interesting result in Table 1 is both volatility measures –risk and absolute return- are very close to each other in terms of their mean and standard deviations.

{Insert Table 1 here}

6. Results

In this section we summarize findings regarding our seven hypotheses in order.

6.1.0 HFTs' order flow toxicity analysis

H₁: *HFTs exert higher order flow toxicity on non-HFT liquidity suppliers than they do on HFT liquidity suppliers.*

We measure the order flow toxicity with the VPIN measure that is developed by Easley, Prado, and O'Hara (2012-a). We calculate VPIN for trades in 120 stocks in six different samples, which are categorized according to the trade initiator, or liquidity seeking side, of the trade during 2009. Our six samples include pure HFT trades (HH), pure non-HFT trades (NN), trades where one side is an HFT (non-HFT) while the other side is a non-HFT (HFT), i.e. HN and NH samples, and all HFT initiated trades (HHHN), all non-HFT initiated trades (NNNH),

By applying probability of informed trading (PIN) measure, Easley et al. (1996) find that high volume stocks have lower probability of informed trading and vice versa. The authors explain this finding by different arrival rates of informed trading in different volume deciles. In the spirit of Easley et al., we measure order flow toxicity for overall sample as well as three different volume deciles. Each decile consists of forty stocks, which are sorted according to their number of shares traded in 2009.

Figure 1 panel A illustrates the distribution of the toxicity in the overall sample and in samples sorted according to volume. Figure 1 shows the highest toxicity is in the pure HFT trades (HH) in the

overall, low volume, and medium volume samples. Regardless of trader type (HFT or non-HFT), high volume stocks have the lowest toxicity. When pure HFT trades (HH) are compared with pure non-HFT trades (NN), pure HFT trades seem to have higher toxicity than the pure non-HFT trades, except in the high volume sample. Another notable point in this figure is that all HFT initiated trades (HHHN) have higher toxicity than the all non-HFT initiated trades (NNNH) in the overall sample and across all volume sorted samples except the high volume sample.

{Insert Figure 1 here}

Table 2 summarizes the mean, median, and standard deviation of order flow toxicity measure in overall sample, samples sorted according to volume (columns) and samples created according to trade initiators (rows). To illustrate the behavior of order flow toxicity in HFT and non-HFT initiated trades; we calculate the cumulative probability distributions of four samples. The results are illustrated in Figure 2 Panel A and B. Figure 2 Panel A represents cumulative probability distributions of order flow toxicity in the pure HFT and pure non-HFT trades. Figure 2 Panel B illustrates cumulative probability distributions of order flow toxicity in all HFT initiated trades and all non-HFT initiated trades. These two panels vividly illustrate that distribution of non-HFT trades cross that of HFT trades at one point, and lie on the left order flow toxicity distribution of HFT trades for VPIN values greater than 0.4. The Figure 2 hints that the order flow toxicity in HFT samples and non-HFT samples are different.

{Insert Table 2 here}

{Insert Figure 2 here}

To test the statistical significance of the observations presented in Figure 1 and Figure 2 we run several statistical tests, namely; the Kruskal-Wallis test and the Mann-Whitney test (also called the Wilcoxon rank sum test). We select these tests by following Easley et al. (1996) approach. Specifically, since the VPIN measure is restricted between zero and one, normality required for standard statistical tests may be violated. The Kruskal-Wallis test determines whether distributions of VPIN are identical over seven different samples (i.e. overall, HH, NN etc…) and over three different volume deciles (i.e. 1st,

2nd and 3rd). These test statistics are given in Table 3, Panel A and B. The Wilcoxon test allows us to compare samples in pairs. Specifically, we test whether VPIN values in one sample higher or lower than another sample. These test statistics are given in Table 3, Panel C and D.

First, we test whether order flow toxicity is different in three volume deciles, by comparing the 2nd, 3rd, and 4th columns of Table 2. According to Easley et al. (1996) lower arrival rates of uninformed traders to low volume stocks increases risk of informed trading in low volume stocks. The Kruskal-Wallis test, results reported in Table 3 Panel A, strongly rejects the hypothesis that order flow toxicity distributions in the three volume samples are same. Table 3 Panel A results show that volume affects the order flow toxicity in overall sample, and all sub-samples created according to trade initiator. To compare the samples pairwise we apply the Mann-Whitney test, results are summarized in Table 3 Panel C. Table 3 Panel C results show that low volume stocks have the highest risk of informed trading, it is followed by medium volume and high volume stocks. These findings are consistent with Easley et al. (1996) findings.

{Insert Table 3 here}

Second, we test if trade initiator type matters in order flow toxicity, by comparing 2nd -7th rows of Table 2. The Kruskal-Wallis test, Table 3 Panel B, shows that samples created according to trade initiators have different distributions. To compare samples pairwise we apply Wilcoxon rank sum test whose results are summarized in Table 3 Panel D.

We first discuss our results in the overall sample. We find that toxicity of pure HFT (HH) trades is 40% higher than the toxicity of pure non-HFT (NN) trades. When an HFT demands liquidity from a non-HFT (HN), flow toxicity decreases by 22% compared to pure HFT trades (HH). On the other hand when non-HFTs demand liquidity from an HFT (NH) the order flow toxicity increases by 24% compared to pure non-HFT trades (NN). These findings support the view that HFTs may exert toxicity on non-HFT traders.

We find interesting results when we study toxicity in the volume sorted samples. The relations found in overall sample are exaggerated in the low volume sample. The pure HFT trades' (HH) toxicity is 75% higher than the pure non-HFT trades (NN). All HFT initiated trades are nearly 35 % more toxic than the all non-HFT initiated trades. When HFTs demand liquidity from non-HFTs (HN) the toxicity decreases by 25% compared to pure HFT trades (HH). On the other hand when non-HFTs demand liquidity from HFTs (NH) the order flow toxicity increases nearly by 57% compared to pure non-HFT trades (NN). These findings show that the HFT toxicity on non-HFTs is a more problematic in low volume stocks. The medium volume statistics are similar to the findings in overall sample.

The relations that hold in overall, low volume, and medium volume samples change in the high volume sample. Interestingly, pure non-HFT trade toxicity is higher than that of pure HFT trades by 53% in the high volume sample. The toxicity of all non-HFT initiated trades is higher than that of all HFT initiated trades by 30%. When HFTs demand liquidity from non-HFTs they experience 17% higher toxicity compared to pure HFT trades. When non-HFTs demand liquidity from HFTs they experience a 26% decrease in toxicity compared to pure non-HFT trades.

Overall, our results show that in the overall, medium volume, and more notably, in low volume samples HFTs exert a significant amount of order flow toxicity on non-HFTs. On the other hand, in high volume, HFTs decrease the toxicity in non-HFT trades. The analysis shows that the overall sample experiences higher order flow toxicity due to HFTs even though HFTs are beneficial in high volume stocks. The detrimental effect of HFTs in the overall sample is mainly due to the high toxicity caused by HFTs in low volume and medium volume stocks.

These findings support our fist hypothesis, implying HFTs may exert higher order flow toxicity to non-HFT liquidity suppliers than to HFT liquidity suppliers. These findings are consistent with theoretical predictions of Cartea and Panelva (2011), that HFTs may cause losses to liquidity traders; of Biais, Foucault, and Moinas (2011), that high levels of HFT can impose adverse selection costs on slow traders and of Jarrow and Protter (2011), that HFTs may have a dysfunctional role in markets. Our findings also

support the empirical findings of Brogaard, Hendershott, and Riordan (2012) that when HFTs provide liquidity with non-marketable orders they may damage non-HFT liquidity suppliers.

6.1.0.1 A possible explanation for HFT induced toxicity

To provide a possible explanation for difference in HFT induced flow toxicity in different volume samples, we develop following argument. First following the high frequency trading literature we make three basic assumptions.

Assumptions:

1- There are two sorts of traders in the market HFT and non-HFTs.

2- HFTs have a speed advantage; they can react to a signal faster than non-HFTs (i.e. Jarrow and Protter, 2011).

3- HFTs tend to revert their positions to a mean of about zero in very short time (Kirilenko et al., 2012).

We examine the flow toxicity for a given level of volume ($V = \tau$) for two cases, namely; with HFTs and without HFTs. We compare the VPINs in a single volume bucket size of (τ) for two cases.

Case 1: Assume that non-HFT buyer will buy $(1 - 2\alpha)\tau$ shares and HFTs will buy $(\alpha\tau)$ shares. HTFs have a speed advantage and so they will buy $(\alpha\tau)$ shares at ($V = 0$), then non-HFTs buyer will buy $(1 - 2\alpha)\tau$ shares. Since HFTs don't hold their positions for long periods of time, HFTs revert their position to zero and sell $(\alpha\tau)$. At this point volume bucket is filled (i.e. $V = \tau$).

By definition

$$VPIN_1 = \frac{(1 - 2\alpha)\tau + \alpha\tau - \alpha\tau}{\tau}$$
$$VPIN_1 = (1 - 2\alpha)$$

Case 2: Assume that non-HFT buyer will buy $(1-2\alpha)\tau$ shares and there are no HFTs in the market. In case two, a certain non-HFT will buy $(1-2\alpha)\tau$ shares at $(V=0)$. The remaining $(2\alpha\tau)$ volume will be filled by trading (buying and selling) activities of non-HFTs. Let a non-HFT buy $(\beta\tau)$ shares and a non-HFT sell $(2\alpha-\beta)\tau$ shares. At this point volume bucket is filled (i.e. $V=\tau$).

By definition

$$VPIN_2 = \frac{(1-2\alpha)\tau + \beta\tau - (2\alpha-\beta)\tau}{\tau}$$
$$VPIN_2 = (1-4\alpha+2\beta)$$

By comparing $VPIN_1$ and $VPIN_2$, we see that flow toxicity will be higher in the volume buckets with HFTs as long as $(\alpha>\beta)$. Implying, in a fixed volume with a fixed amount non-HFT initiated trades, when the fraction of HFT trade is greater than non-HFT trades, toxicity will be higher. We can interpret this result with accordance Easley et al. (1996). Since the arrival rates of non-HFT trades different in low volume and high volume stocks, HFTs fraction of trades that is matched with non-HFT trades differs with the volume of stock. As a result HFTs cause higher flow toxicity in some deciles while decrease order toxicity in others.

6.1.1 Determinants of order flow toxicity, VPIN metric.

H₂: *Trade intensity and return volatility are significant determinants of the order flow toxicity measure VPIN.*

By definition of Easley et al. (2012-a) the toxicity measure VPIN is based on order imbalances and trade intensity. Accordingly, we expect trade characteristics such as trade size and number of trades to be significant determinants of the metric. Easley et at. also argue that order flow is considered to be toxic when it causes losses to liquidity suppliers. Griffiths et al. (2000) argues that liquidity suppliers' willingness is affected by the price volatility. Depending on both arguments, we expect price volatility to be an important determinant of the VPIN measure.

Our formal model is:

$$\ln VPIN_{\tau} = c_0 + c_1 \ln Volatility_{\tau-1} + c_2 \ln No.Trades_{\tau-1} + c_3 \ln TradeSize_{\tau-1} + e_{\tau},$$

Detailed explanations for variable calculation procedures are given in section 2.2. For volatility we use two proxies from literature; namely absolute return proxy from Easley et al. (2012) and average return volatility based on volume clock. We normalize our variables following the Easley et al. (2008) by taking their natural logarithms.

{Insert Table 4 about here}

Table 4 Panel A summarizes the correlation between the proposed determinants of VPIN and the metric itself. Similar to Easley et al. (2012-a) findings, we find that volatility measures are positively correlated to the order flow toxicity in subsequent volume bucket. On the other hand number of trades and average trade size per trade in each bucket are negatively correlated to order flow toxicity in subsequent volume bucket.

Table 4 Panel B summarizes the results of four different models. In model 1 and 2 we use two different volatility measures interchangeable. Also, to test if the relation between the trade intensity measures and VPIN metric, we include squares of the variables in these two models. In both models all of our control variables are statistically significant at 0.01 levels. We find that while volatility increases the flow toxicity, the trade intensity decreases it to some degree. However, the relation between trade intensity and VPIN metric is a non-linear one, since the square terms are statistically significant. Model 3 and 4 assumes a linear relationship between trade intensity and order flow toxicity, though all determinants are still significant the explanatory powers of the models are lower compared to model 1 and 2. Overall, our results show that average trade size per trade, volatility, and numbers of trades are important determinant of order flow toxicity.

6.1.2 Relation between Risk, Absolute Return and VPIN Metric

H₃: *The order flow toxicity in volume bucket $\tau - 1$ will be related to volatility in volume bucket τ.*

Easley et al. (2012-a) hypothesize that "*Persistently high levels of VPIN lead to volatility.*" The authors examine the correlation and conditional probabilities of $VPIN_{\tau-1}, and \mid \frac{P_\tau}{P_{\tau-1}} - 1 \mid$. This examination is done in volume time and they find supportive evidence for their hypothesis. On the other hand, Andersen and Bondarenko (2013), by calculating the average absolute one minute returns (AAR) over five minutes and one day periods, find that the VPIN measure is negatively related with AAR. Andersen and Bondarenko conclude that VPIN does not have any incremental forecasting power for future volatility. Both Easley et al. and Andersen and Bondarenko studies are conducted by using E-mini S&P 500 futures contract data. In this section, we examine the volatility and VPIN relation by using equity market data. Also, since our data indicates the trade initiator our VPIN measure is free from any trade classification errors.

Our formal model is:

$$\ln Volatility_\tau = c_0 + c_1 \ln VPIN_{\tau-1} + c_2 \ln No.Trades_{\tau-1} + c_3 \ln TradeSize_{\tau-1} + c_4 \ln Volatility_{\tau-1} + e_\tau$$

We calculate two volatility measures; first one is absolute return which is similar to Easley et al. Easley et al. (2012-a), and the second one is risk which is standard deviation of returns over ten sub-volume buckets in each volume bucket. Detailed explanations for variable calculation procedures are given in section 2.2. In the spirit of Easley et al. (2008) we normalize our VPIN metric and other variable by taking their natural logarithms. Our model tests if VPIN has any incremental predictive power after controlling for trade intensity factors and lagged volatility.

{Insert Table 5 about here}

Table 5 Panel A summarizes correlations between volatility in volume bucket τ, and VPIN and trade intensity in volume bucket $\tau-1$. Consistent with Easley et al. (2012) we find that VPIN is positively correlated with two different volatility measures. On the other hand, trade intensity factors are negatively correlated with risk in returns. Table 5 Panel B reports the results of OLS three different regressions, in these models our volatility proxy is risk. In the first model we don't control for trade intensity and find that VPIN is positively related to risk. In the second model after controlling for trade size we find that VPIN has positive predictive power for volatility in subsequent volume bucket. In the third model we control for two trade intensity factors and lagged volatility, and our results in previous models still holds. However, due to the high correlation between number of trades and VPIN, the results in model 3 must be interpreted with caution.

Table 5 Panel C proxies the volatility with absolute return measure as in Easley et al. (2012-a). In model 1 test the predictive power of the VPIN individually, and finds that VPIN has a positive relation with volatility. Model 2 controls for lagged absolute return and model 3 controls for trade intensity and lagged absolute return. Though different specifications all three models show that VPIN is positively related to volatility even after controlling for lagged absolute returns and trade intensity. By comparing R-squares model 2 and model 4, we find that VPIN's explanatory power is higher than the combined two trade intensity factors.

Overall, Table 5 results provide supportive evidence that VPIN metric has positive relation with two different volatility measures, even after controlling for trade intensity. Moreover, VPIN's individual explanatory power is higher than the trade intensity measures.

6.1.3 Protection against Order Flow Toxicity, FVPIN future contracts

H$_4$: *FVPIN futures contracts provide positive returns to investors*

Order flow is toxic if it causes loses to liquidity providers (Easley et al., 2012-a). We analyze the toxicity in a high frequency world for two trader types i.e. HFT and non-HFT. In this section we examine

a futures contract, FVPIN, which is developed by Easley, Prado, and O'Hara (2011). According to Easley, Prado, and O'Hara securitization of toxicity measures with a contract such as FVPIN may provide insurance or a hedging opportunity to liquidity providers against toxicity. This contract is valued as [-ln(VPIN)] and must be cash-settled on a daily basis.

We test the protection power of the FVPIN contract by using the VPIN calculation of 120 stocks, HFT and non-HFT trade data of these stocks is provided by NASDAQ throughout 2009. We employ a basic strategy in which, the investor purchases the contract at the beginning of the day, by paying $[P_0=-\ln(VPIN_{open})]$, and sells the contract at the end of session at $[P_1=-\ln(VPIN_{close})]$. We calculate the annual average daily percentage return of this strategy for our overall sample, high volume, medium volume and low volume stocks.

{Insert Table 6 about here}

Table 6 reports the returns of FVPIN contracts for four different samples. Average daily return of FVPIN contract is 2.87%, implying that, for overall sample, the FVPIN contract may, on average, protect against toxicity. When we divide the overall sample into volume groups, we find that FVPIN is beneficial in all volume deciles. Specifically, FVPIN provides 4.452%, 2.755% and 1.421 % average returns in low volume, medium volume and high volume stocks, respectively. Though percent return on FVPIN contracts is smallest in the high volume stocks, the standard deviation is also smallest in high volume decile. When we compare coefficient of variations, across all samples, we find that high volume stocks bears the smallest risk for a given level of return. Figure 3 vividly reports the behavior of FVPIN contract returns in three different volume deciles.

{Insert Figure 3 about here}

Overall, in our 120 stocks sample FVPIN provides a hedging opportunity against order flow toxicity in overall sample and three different volume sorted samples. Figure 3 graphically illustrates the FVPIN results for the four samples. It clearly shows that FVPIN may provide a hedging opportunity in

high volume, medium volume, and low volume stocks. However the magnitude and volatility of returns vary across the samples.

6.1.4 HFTs and stock price volatility

The second issue our study focuses on is HFTs' impacts on the stock price variance. When examining this impact we apply a volume based variance calculation approach rather than time based one since Easley, Prado, and O'Hara (2012-a) argue that volume clock is more relevant than time based in a high frequency world. To apply a volume bar based variance calculation, we create volume buckets as in the order flow toxicity (VPIN) analysis in section 3.1. The observed variance is calculated by using the highest price and the lowest price in each volume bucket. We follow the Parkinson (1980) procedure, which defines $\sigma_{HL}^2 = \frac{0.361}{n}\sum_{i=1}^{n}l_i^2$ and $l_i = \ln(\frac{H_i}{L_i}), i = 1,2\ldots n$, to calculate observed variance. Actual variance of stock prices, which is calculated by finding the positive root of (eq.8) with the following formula

$$(\sigma_{H_A L_A})_{1,2} = \frac{-k*c \mp \sqrt{(k*c)^2 - 4*a*(c^2 - a*\sigma_{H_o L_o}^2)}}{2*a} \qquad (eq.9)$$

A detailed explanation of this methodology is given in section 3.2.

6.1.4.1 Volume based observed variance comparison throughout 2009

H5: *HFTs will increase observed stock price variance.*

We calculate the volume based observed variances of six samples in 2009 by using the 120 stocks, the NASDAQ provided data reports HFT and non-HFT trades of these stocks. Table 7 panel A shows that, in the overall sample, pure non-HFT trades' variance is nearly twice of that of the pure HFT trades. Similarly all non-HFT initiated trades have a higher variance than the all HFT initiated trades. When we compare variances (NH) sample and (NN) sample, we find that when providing liquidity to the non-HFTs, HFTs decreases the variance in pure non-HFT trades by 37%. On the other hand, comparing

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(HH) and (HN) samples show that when HFTs demand liquidity from non-HFTs (HN), the observed variance increase by 66% compared to pure HFT trades (HH). Similar variance impacts of HFTs can be found throughout the low volume and medium volume subsamples (Panel B and C). But in high volume stocks variances of samples created according to trade initiators are not statistically different from one another.

{Insert Table 7 about here}

In short, when HFTs demand liquidity from non-HFTs, they increase observed variance. This finding is consistent with theoretical predictions of Cartea and Penalva (2011) and Jarrow and Protter (2011) and empirical findings of Zhang (2010). When HFTs provide liquidity they decrease variance. This result is consistent with the findings of Brogaard (2010), which shows HFTs may reduce the stock price volatility.

6.2.2 Actual versus observed variance comparison

By solving equation 8, we calculate a variance measure that is free from the impact of bid-ask spread bounce, which we call an actual variance measure. We use the quote data for 120 NASDAQ stock for one week in 2010 to calculate the actual variances and observed variances for our sample. Table 8 Panel A compares the actual variance and observed variance results for our one week sample. Throughout all seven samples we find that the actual variance is significantly smaller than the observed one. Without the spread, actual variance is nearly 1.5 % lower than the observed variance in all seven samples.

Table 8 Panel B and Panel C compare the actual and observed variances of six different subsamples. The observed variances in Panel mostly show a statistical difference between the observed variance in samples, consistent with our findings in Table 7. On the other hand, the actual variance results in Panel C show that the differences between the sample variances are insignificant. The differences between Panel B and Panel C support the views of Corwin and Schultz (2012) and O'Hara (2006) that spreads may have an impact on asset prices and accordingly variance. Removing these effects may

produce more accurate results. Another finding from Panel C is that, when impact of the spread is removed, there is actually no significant difference between variances of HFT trades and non-HFT trades.

7. Summary and Conclusion

In this study we examine two important questions related to HFTs by NASDAQ provided HFT dataset. Our first focus is on the order flow toxicity of HFTs to liquidity suppliers. We proxy order flow toxicity by using the VPIN measure of Easley et al. (2012-a). Our results show that all HFT initiated trades always have higher toxicity than the all non-HFT initiated trades in overall sample. The pure HFT trades (HH) have the highest toxicity in all samples except high volume stocks. When comparing pure non-HFT trades (NN), pure HFT toxicity is twice of that of pure non-HFTs. Toxicity problem is more severe in low volume stocks than high volume and medium volume stocks. With these findings we provide supportive empirical evidence to the theoretical predictions of Cartea, and Penalva (2011), Biais, Foucault, and Moinas (2012) and Jarrow, and Protter (2011) that HFTs may play a dysfunctional role in financial markets.

Our examination of the main determinants of flow toxicity reveals that trade intensity and risk of the stock are main determinants of order flow toxicity. Trade intensity variables are negatively related to the order flow toxicity and these relations are non-linear. Risk is positively related to order flow toxicity. In addition we find that in equity markets even after controlling for trade intensity, VPIN has predictive power for future volatility.

We also examine the FVPIN future contract, a hedge tool against flow toxicity which is developed by Easley, Prado and O'Hara (2011). Our results show that FVPIN may be a hedging tool against the toxicity losses for liquidity suppliers in all volume deciles.

Our second focus is on the stock price variance and the HFT activities. Easley, Prado, and O'Hara (2012) argue that in high frequency world volume clock is more relevant than the time clock. Accordingly, in variance calculations we follow a volume based approach to calculate the variance. In addition by building on results of Corwin, and Schultz (2012) and Parkinson (1980), we develop an actual

variance method which eliminates the impact of spread on the observed variance. This measure shows that when impact of spreads is removed actual variance is lower than observed variance. When HFTs demand liquidity from non-HFTs the increase observed variance which is consistent with theoretical predictions of Cartea and Penalva (2011) and Jarrow and Protter (2011) and empirical findings of Zhang (2010). When HFTs provide liquidity they decrease variance this finding is consistent with the findings of Brogaard (2010), which shows HFTs may reduce the stock price volatility.

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Table 1

Descriptive statistics for 120 NASDAQ selected stocks for 2009.

VPIN is the toxicity measure and calculated by flowing Easley et al. (2012). Average price is the mean price per volume bucket. Absolute return is the absolute value of the returns in each volume bucket, and calculated similar to Easley et al. (2012). Risk is the standard deviation of returns in each volume bucket and calculated by dividing each volume bucket into ten sub volume buckets. Trade size is average number of shares traded per trade. Average volume is mean volume bucket size (1/50 of average daily volume of each stock).

Variable	Mean	Min	Max	St.Dev
VPIN	0.332	0.076	1.000	0.150
Average Price	26.611	0.240	625.100	42.070
Absolute Return	0.003	0.000	9.691	0.010
Risk	0.005	0.000	3.711	0.009
Trade size	138.522	0.003	21305.710	122.826
Average Volume	30264.740	121.929	398756.280	67006.000

Table 2

Summary of VPIN Estimate Statistics

This table presents means, medians, and sample standard deviations of VPIN estimates by volume decile and overall samples for the 120 stocks in our sample. The parameter VPIN is a measure of order flow toxicity. 'H' stands for HFT trader, 'N' stands for non-HFT trader. In overall sample (Overall VPIN) trades are not separated. In (HH VPIN) sample the initiator of the trade, liquidity seeking party, is an HFT and the passive side, liquidity supplier, is also an HFT. In (NN VPIN) sample the initiator of the trade, liquidity seeking party, is a non-HFT and the passive side, liquidity supplier, is also a non-HFT. In (NH VPIN) sample the initiator of the trade, liquidity seeking party, is a non-HFT and the passive side, liquidity supplier, is either an HFT. In (HN VPIN) sample the initiator of the trade, liquidity seeking party, is an HFT and the passive side, liquidity supplier, is a non-HFT. In (HHHN VPIN) sample the initiator of the trade, liquidity seeking party, is an HFT and the passive side, liquidity supplier, is either an HFT or non-HFT. In (NNNH VPIN) sample the initiator of the trade, liquidity seeking party, is a non-HFT and the passive side, liquidity supplier, is either an HFT or non-HFT. Three volume deciles are determined according to number of shares traded in year 2009.

	Overall sample	First Decile	Second Decile	Third Decile
Number in sample	120	40	40	40
Overall VPIN				
Mean	0.3323	0.2120	0.3220	0.4630
Median	0.3164	0.2119	0.3224	0.4685
Std. dev.	0.1294	0.0316	0.0718	0.1116
HH VPIN				
Mean	0.6004	0.2184	0.6458	0.9369
Median	0.6686	0.2044	0.6805	0.9651
Std. dev.	0.3214	0.0545	0.1977	0.0727
NN VPIN				
Mean	0.4293	0.3343	0.4195	0.5339
Median	0.4177	0.3313	0.4239	0.5354
Std. dev.	0.1156	0.0534	0.0656	0.1143
NH VPIN				
Mean	0.5347	0.2459	0.5205	0.8378
Median	0.4983	0.2415	0.4983	0.8647
Std. dev.	0.2666	0.0350	0.1518	0.1120
HN VPIN				
Mean	0.4673	0.2645	0.4278	0.7096
Median	0.4095	0.2613	0.4130	0.7775
Std. dev.	0.2299	0.0569	0.1283	0.1937
HHHN VPIN				
Mean	0.4280	0.2104	0.3867	0.6868
Median	0.3791	0.2043	0.3793	0.7424
Std. dev.	0.2398	0.0454	0.1253	0.1967
NNNH VPIN				
Mean	0.3885	0.2748	0.3801	0.5105
Median	0.3766	0.2734	0.3842	0.5016
Std. dev.	0.1220	0.0324	0.0642	0.1077

Table 3

Nonparametric Tests

The Kruskal-Wallis null hypothesis is that parameter values for all three volume samples and trader samples are drawn from identical populations. The alternative hypothesis is that at least one of the populations has greater observed values than other populations. The VPIN variable is measuring order flow toxicity. The Mann-Whitney test null hypothesis is that two samples are drawn from identical populations. Its alternative hypothesis is that one population yields higher values. The VPIN variable is measuring order flow toxicity.

Panel A: Kruskal-Wallis test on VPIN by Volume	
Sample	Test statistic
Overall	87.4202
NN	67.8367
HH	99.1013
HN	84.3294
NH	99.7731
NNNH	87.3521
HHHN	90.2591

Panel B: Kruskal-Wallis test on VPIN by Trader Type	
Test Statistic	64.7727
Critical Value for $\alpha = 0.05$ is 5.991	

Panel C: Mann-Whitney Tests on VPIN			
Pairwise Comparisons (n=40, m=40)			
	Low vol. to High Vol.	Med vol. to High Vol.	Low vol. to Med vol.
Overall	7.6355	48.2670	5.6725
NN	7.0870	5.6340	4.7872
HH	7.6932	7.5777	6.7887
HN	7.5585	6.3942	5.9515
NH	7.6932	7.5296	6.9523
NNNH	7.6451	6.9234	5.6628
HHHN	7.6739	6.8849	6.1632

Panel D: Wilcoxon-Mann-Whitney Test				
	Overall	First decile	Second decile	Third decile
HH vs. NN	2.9576	-6.7598	4.7872	7.6162
HH vs. HN	2.4964	-4.3349	4.5947	6.0766
NH vs. NN	1.6689	-6.6540	3.0151	7.0485
HHHN vs. NNNH	-0.5662	-6.3749	-0.1780	4.0270

The test statistic is normally distributed and the critical value for $\alpha=0.05$ is ± 1.6449

Table 4

Determinants of VPIN

VPIN (τ) is the toxicity in the volume bucket (τ). Risk (τ-1) is the standard deviation of returns in volume bucket (τ-1) and calculated by dividing the volume bucket into ten equal sub volume buckets. No. of trades (τ-1) is the number of trades occurred in volume bucket (τ-1). Trade size (τ-1) is the average number of shares traded in volume bucket (τ-1). Return (τ-1) is the absolute return in volume bucket (τ-1) and calculated similar to Easley et al. (2012). In Panel B, all models are OLS models. Model 1 and model 2 full models. Model 3 and model 4 drops the squares of the trade intensity variables.

Panel A: Pearson Correlation Coefficients (Rho)					
	VPIN(τ)	Risk(τ-1)	No. of Trades(τ-1)	Trade size (τ-1)	Return (τ-1)
VPIN(τ)	1.0000				
Risk(τ-1)	0.3919	1.0000			
No. of Trades(τ-1)	-0.6941	-0.4324	1.0000		
Trade size (τ-1)	-0.2821	-0.1724	-0.0922	1.0000	
Return (τ-1)	0.0897	0.2955	-0.0169	-0.0540	1.0000
Panel B: Multivariate Analysis of determinants of VPIN					
VPIN(τ)	Model 1	Model 2	Model 3	Model 4	
Risk(τ-1)	0.01698		0.00821		
Return(τ-1)		0.00580		0.012753	
No. of Trades(τ-1)	-0.40566	-0.39794	-0.17226	-0.17442	
No. Trades(τ-1)^2	0.02575	0.02434			
Trade size(τ-1)	-0.10774	-0.11684	-0.14887	-0.14949	
Trade size(τ-1)^2	0.00997	0.01031			
Cons	0.28598	0.25553	0.21561	0.26663	
R-Squared	0.6824	0.6814	0.6035	0.6060	

All coefficients are significant at 0.01 levels.

Table 5

Relations between Risk, Return and VPIN

Risk (τ) is the standard deviation of returns in volume bucket (τ) and calculated by dividing the volume bucket into ten equal sub volume buckets. Return (τ) is the absolute return in volume bucket (τ) and calculated similar to Easley et al. (2012). VPIN (τ-1) is the toxicity in the volume bucket (τ-1). Volume (τ-1) is the number of trades occurred in volume bucket (τ-1). In Panel B, all models are OLS models with dependent are variable as risk. Model 1 measures the predictive power of VPIN after controlling only lagged risk. Model 2 tests impact of VPIN on risk, after controlling one trade intensity variable and lagged risk variable. Model 3 includes all trade intensity variables and lagged risk variable. In panel C all models are OLS models with dependent variable as absolute return. Model 1 shows the individual predictive power of VPIN metric. Model 2 controls for lagged absolute return while model 3 controls for lagged absolute return and trade intensity proxy. Model 4 measures explanatory power of trade intensity for volatility after controlling lagged absolute return.

Panel A: Pearson Correlation Coefficients (Rho)					
	Risk(τ)	Abs. Return (τ)	VPIN(τ-1)	Trade size(τ-1)	No. of Trades(τ-1)
Risk(τ)	1.0000				
Abs. Return (τ)	0.2919	1.0000			
VPIN(τ-1)	0.3992	0.0830	1.0000		
Trade size(τ-1)	-0.1668	-0.0326	-0.2823	1.0000	
No. of Trades(τ-1)	-0.4436	-0.0226	-0.6956	-0.0816	1.0000
Panel B: Risk and VPIN Multivariate Analysis					
Risk(τ)	Model 1		Model 2		Model 3
VPIN(τ-1)	0.29133		0.28700		0.04593
Risk(τ-1)	0.70849		0.70794		0.67817
No. of Trades(τ-1)					-0.08295
Trade size(τ-1)			-0.00757		-0.05445
Constant	-1.38151		-1.3544		-1.27649
R-Squared	0.5818		0.5818		0.5892
Panel C: Absolute Return and VPIN Multivariate Analysis					
Return(τ)	Model 1	Model 2	Model 3		Model 4
VPIN(τ-1)	0.38157	0.382615	0.26313		
Return(τ-1)		0.25753	0.25688		0.26030
No. of Trades(τ-1)					-0.04155
Trade size(τ-1)			-0.02452		-0.02533
Constant	-6.38574	-4.7420	-4.6557		-4.77325
R-Squared	0.0081	0.0741	0.0743		0.0699

All coefficients are significant at 0.01 levels.

Table 6: FVPIN Contract Annual Returns Analysis

Value of FVPIN contract is defined as [-ln(VPIN)], we assume the investor buys the contract at the beginning of the day, and sells it at the end of each day throughout 2009. The VPIN measure is the overall VPIN calculated for 120 select NASDAQ stocks in 2009. Table-6 summarizes mean, median and standard deviation of average (%) return of each sample when an investor follows the above defined trading strategy. Coefficient of variation is ratio of standard deviation to mean.

	Mean	Median	Standard Deviation	Coefficient of Variation
Overall Sample	2.876%	2.328%	2.165%	0.7529
Low Volume	4.452%	4.018%	2.779%	0.6242
Medium Volume	2.755%	2.568%	2.779%	1.0080
High Volume	1.421%	1.341%	0.579%	0.0004

Table 7: Volume based observed variance comparison throughout 2009

Mean_1 refers to mean observed variance of the first sample in each compared pair. Mean_2 refers to the second sample in compared pair. Difference equals to Mean_1 minus Mean_2. All values are multiplied by 100. T-stat. and P-value are the statistics of T-test that tests the null hypothesis of difference equals to zero.*, **, *** are significant at 10%, 5% & 1% levels respectively. The sample period is the year 2009.

	Mean_1	Mean_2	Difference	P-value
Panel A: Overall Sample				
HH vs. NN	0.234	0.512	-0.278***	0.0001
HH vs. HN	0.234	0.388	-0.154***	0.0001
NH vs. NN	0.322	0.512	-0.190***	0.0001
HHHN vs. NNNH	0.351	0.652	-0.300***	0.0001
Panel B: Low Volume				
HH vs. NN	0.057	0.638	-0.580***	0.0001
HH vs. HN	0.057	0.328	-0.271***	0.0001
NH vs. NN	0.186	0.638	-0.452***	0.0001
HHHN vs. NNNH	0.351	0.652	-0.300***	0.0001
Panel C: Medium Volume				
HH vs. NN	0.335	0.592	-0.257***	0.0001
HH vs. HN	0.335	0.559	-0.223***	0.0001
NH vs. NN	0.494	0.592	-0.098*	0.0789
HHHN vs. NNNH	0.540	0.541	-0.001	0.9939
Panel D: High Volume				
HH vs. NN	0.309	0.306	0.003	0.9392
HH vs. HN	0.309	0.278	0.030	0.4123
NH vs. NN	0.286	0.306	-0.020	0.5837
HHHN vs. NNNH	0.222	0.223	-0.002	0.9542

Table 8: Actual versus Observed variance

Table 8 Panel A compares differences between observed variance, calculated as a classical way, and actual variance, calculated by using the measure we developed and considers the impact of the spreads on the observed variance. Panel B compares differences between observed variances of six samples. Panel C compares differences between actual variances of six samples. The sample period is as the week of Feb 22 – 26, 2010.

Panel A: Overall sample				
	Observed Var.	Actual Var.	Diff.(%)	P-value
Overall	0.6015	0.5881	1.34***	0.0001
HN	0.6013	0.5882	1.31***	0.0001
NH	0.6013	0.5868	1.46***	0.0001
HH	0.6012	0.589	1.23***	0.0001
NN	0.6014	0.586	1.46***	0.0001
NNNH	0.6014	0.5872	1.42***	0.0001
HHHN	0.6013	0.5887	1.26***	0.0001
Panel B: Observed variance comparison of subsamples				
	Mean_1	Mean_2	Diff.(%)	P-value
HH vs. NN	0.6012	0.6014	-0.017***	0.0001
NH vs. NN	0.6013	0.6014	-0.007	0.1083
HH vs. HN	0.6012	0.601	-0.008**	0.0472
HHHN vs. NNNH	0.6013	0.6014	-0.011**	0.0146
Panel C: Actual variance comparison of subsamples				
	Mean_1	Mean_2	Diff.(%)	P-value
HH vs. NN	0.5890	0.5868	0.2160	0.3840
NH vs. NN	0.5868	0.5868	-0.0030	0.9910
HH vs. HN	0.5890	0.5882	0.0722	0.7696
HHHN vs. NNNH	0.5887	0.5872	0.1470	0.5486

Figure 1

This figure shows the VPIN in the overall sample and three samples sorted by volume. VPIN measure is calculated following Easley et al. (2012-a) methodology. The samples consist of 120 select NASDAQ stocks for the entire year of 2009. Volume is the number of shares traded in 2009.
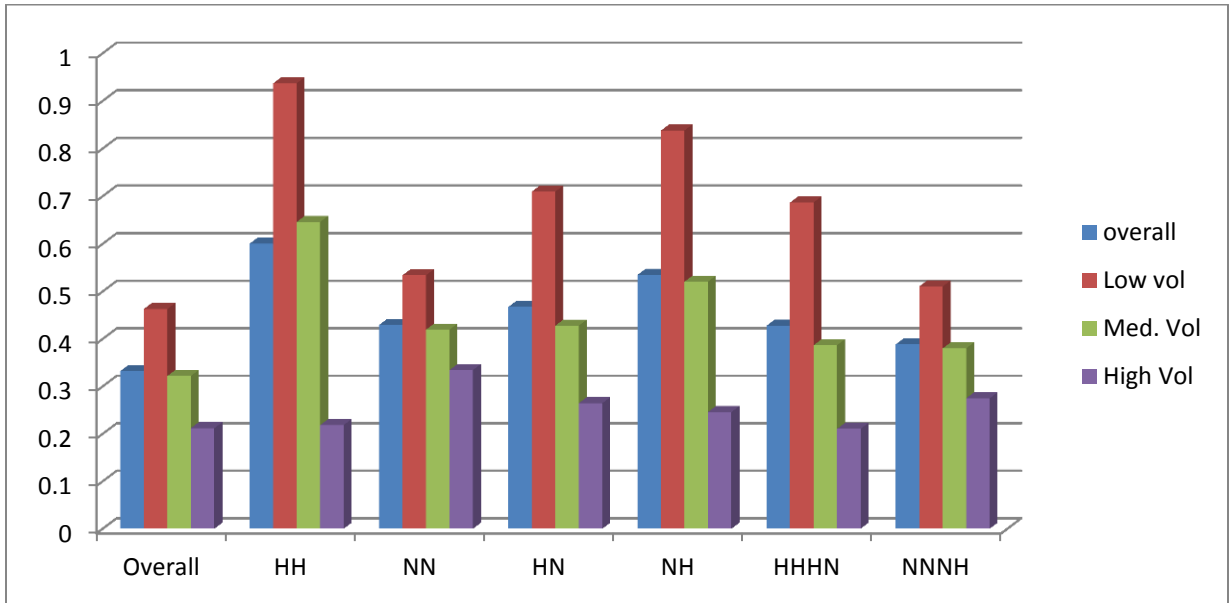
Figure 2

Figure 2 Panel A shows cumulative probability distributions of VPIN metric in pure HFT trades and pure non-HFT trades. Figure 2 Panel B shows cumulative probability distributions of VPIN metric in all HFT initiated trades and all non-HFT initiated trades. Sample is the same as the one in Figure 1.
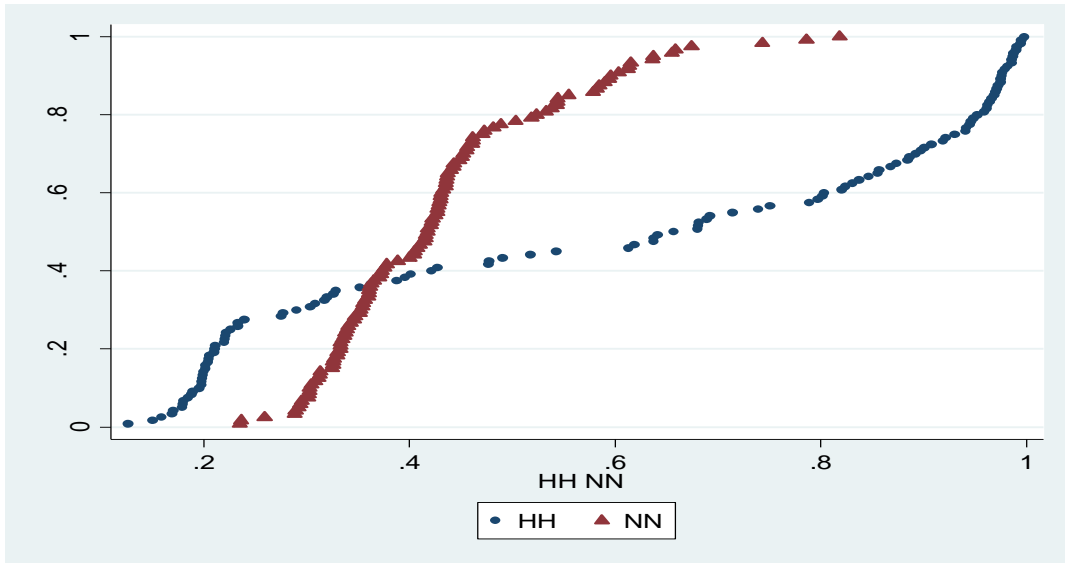
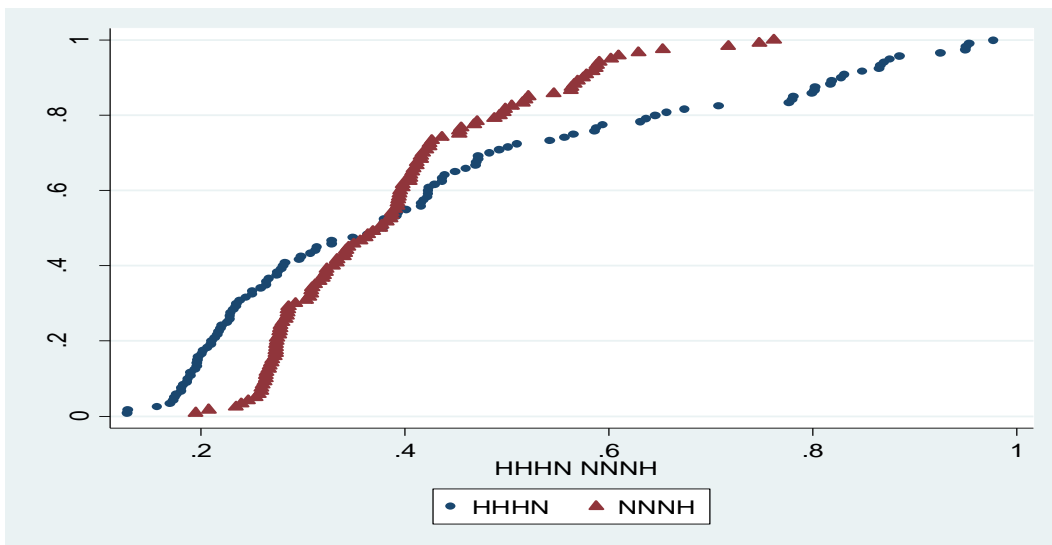**Figure 2 Panel A**

Figure 2 Panel B

Figure 3

Value of FVPIN contract is defined as [-ln(VPIN)], we assume the investor buys the contract at the beginning of the day, and sells it at the end of each day throughout 2009. The VPIN measure is the overall VPIN calculated for 120 select NASDAQ stocks in 2009. Figure shows the average FVPIN contract returns for three volume deciles. Volume is the number of shares traded in 2009.