Creating Controversy in Proxy Voting Advice*

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Abstract

The quality of proxy advisors' voting recommendations is important for policymakers and industry participants. We analyze the design of recommendations (available to all market participants) and research reports (available only to subscribers) by a proxy advisor, whose objective is to maximize its profits from selling information to shareholders. We show that even if all shareholders' interests are aligned and aim at maximizing firm value, the proxy advisor benefits from biasing its recommendations against the *a priori* more likely alternative. Such recommendations "create controversy" about the vote, increasing the probability that the outcome is close and raising each shareholder's willingness to pay for advice. In contrast, it serves the interest of the proxy advisor to make private research reports unbiased and precise. Our results help reinterpret empirical patterns of shareholders' voting behavior and suggest that proxy advisors' recommendations may not be a suitable benchmark for evaluating the votes of asset managers. They also rationalize the one-size-fits-all approach in proxy advisors' recommendations.

Keywords: proxy advisor, voting, sale of information, information design, Bayesian persuasion, controversy, bias, corporate governance, one-size-fits-all approach

JEL classifications: D72, D82, D83, G34, K22

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1 Introduction

Proxy advisory firms have emerged as major players in corporate governance. They make recommendations on how to cast votes and provide research reports for subscribing shareholders that contain the rationales for recommendations, including information on various aspects of firms' governance practices. While proxy advisors' research reports are only available to their subscribing shareholders, their recommendations are often made public through the media. Through both these public recommendations and private research reports, proxy advisors such as ISS in particular, have a substantial impact on voting outcomes (e.g., Alexander et al., 2010; Iliev and Lowry, 2015; Malenko and Shen, 2016).

Given the strong influence of proxy advisors, the quality and information content of their research reports and recommendations have become an important topic of discussion among market participants and policymakers. For example, over 2018–2020, the SEC adopted a number of regulatory changes to ensure "that investors who use proxy voting advice receive more transparent, accurate, and complete information on which to make their voting decisions."[1] According to the survey evidence in McCahery, Sautner, and Starks (2016), there is heterogeneity among institutional investors in their views about proxy advisors' research: while 55% of the respondents believe that proxy advisors help them make more informed voting decisions, 30% are concerned that the resulting advice is too standardized.

Do proxy advisors have incentives to produce unbiased and informative research and recommendations? In particular, as the SEC's Concept Release on the U.S. Proxy System stated, "Does the lack of a direct pecuniary interest in the effects of their recommendations on shareholder value affect how they formulate recommendations," "what criteria and processes do proxy advisory firms use," and are their "proxy research reports ... materially accurate and complete"?[2] Motivated by these questions, this paper studies the information design problem of a proxy advisor who aims to maximize its profits from information sale to voters.

Our main result is that even if all shareholders are unbiased and aligned at maximizing the value of their shares, the profit-maximizing proxy advisor often has incentives to produce *public recommendations that are biased* against the *a priori* more likely alternative. By recommending for the unexpected alternative too frequently, it "creates controversy" around

[1] See, e.g., https://www.sec.gov/rules/interp/2019/34-86721.pdf and https://www.sec.gov/rules/final/2020/34-89372.pdf. In November 2021, the SEC proposed rescinding two rules adopted in 2020, given the concern that they might "potentially impair the independence and quality of the proxy voting advice" (see https://www.sec.gov/news/press-release/2021-236).

[2] See p. 125 in https://www.sec.gov/rules/concept/2010/34-62495.pdf.

the proposal and increases the probability that the vote will be close. At the same time, the advisor has incentives to produce informative and *unbiased research reports* for its subscribers. These results suggest a reinterpretation of the empirical evidence on shareholders' voting patterns and proxy advisory recommendations.

In our model, shareholders vote on a proposal whose value depends on the unknown state, and the proxy advisor faces an information design problem, modeled as Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). Specifically, it designs two signals about the state: one, which we refer to as the "research report," is available only to its subscribers, and the other, which we refer to as the "voting recommendation," is publicly observed by all shareholders. In addition, the advisor sets a fee for subscribing to its research report. Each shareholder decides whether to pay the fee and get the report or whether to only observe the public voting recommendation. The state is then realized, the proxy advisor's report and recommendation are produced, and shareholders cast their votes based on the information they receive. The proposal is approved if it receives a majority of the votes.

This setup corresponds to the observed voting practices. Prior to the shareholder meeting, proxy advisors deliver to their subscribers a research report with details about the company's governance practices and considerations about the proposals on the agenda. In addition, for each proposal on the agenda, the report contains a recommendation on whether to vote in favor or against this proposal. Importantly, while research reports are only available to the subscribers, the recommendations, especially for contentious meetings in which proxy advisors recommend against management on certain proposals, are typically shared by the media and are therefore publicly available to all shareholders.[3] One potential way to interpret the advisor's design of recommendations are the voting guidelines, which proxy advisors revise each year and announce publicly. These guidelines describe, for various types of proposals, detailed rules and criteria that the advisor plans to use when making recommendations for each individual company and proposal.[4] Proxy advisors tend to follow such guidelines because of both regulatory and reputational costs of deviating.[5] However, the

[3] See, e.g., "ISS urges UniCredit investors to reject Orcel's pay package" (March 30, 2021), *Reuters*.

[4] See, e.g., https://www.issgovernance.com/policy-gateway/voting-policies/ for ISS' voting guidelines, and https://www.glasslewis.com/voting-policies-current/ for Glass Lewis' guidelines.

[5] For example, the 2019 SEC's guidance for proxy advisors (https://www.sec.gov/rules/interp/2019/34-86721.pdf) states that to avoid a potential violation of Rule 14a-9, proxy advisors "may need to disclose ... an explanation of the methodology used to formulate its voting advice on a particular matter (including any material deviations from the provider's publicly-announced guidelines, policies, or standard methodologies for analyzing such matters)."

information design problem of the advisor should be understood more generally than just the design of the guidelines. Importantly, as we discuss in more detail in the paper, the chosen information design policies are time-consistent: because the advisor does not obtain any ex-post benefit from a vote outcome in either direction, and simply maximizes its ex-ante profit from information sales, it has no ex-post incentives to deviate from the design of either the research report or recommendations.

While the proxy advisor designs the recommendations and research reports to maximize its profit from information sales, it does not maximize the value of either the asset managers or the operating companies, highlighting a fundamental conflict of interest. In fact, value maximization for the downstream entities (asset managers and operating companies) would suggest that the public recommendations are perfectly aligned with the reports and reveal all the information the proxy advisor has—but then no investors would need to purchase the reports and the proxy advisor would have zero revenue. (Arrow (1962) highlights the underlying challenge of getting paid for information.)

This raises the question of which design of recommendations and reports would maximize the fees that can be obtained from the subscribers. There is no obvious answer to this problem. For example, it may be natural to expect that the advisor will produce totally uninformative public recommendations, so as not to dilute the value of the private reports, and only give informative signals for a fee. Or, to the extent that public recommendations reveal some information, it may be natural to expect that they will be unbiased since all shareholders are unbiased and aligned at maximizing firm value.

Nevertheless, we show that the proxy advisor's profit from information sale is maximized if (1) it designs a fully informative and unbiased research report, and (2) provides a public recommendation that is partially informative but biased against the alternative that is *a priori* more likely to be value-increasing (as long as the *a priori* likelihood of it being value-increasing is high enough). We refer to this bias against the more likely alternative as "creating controversy." The key idea behind such information design is that the advisor manipulates the public signal in order to increase each shareholder's willingness to pay for the private signal.

To see the intuition, consider one of the most frequent and important issues on which shareholders vote: the approval of directors proposed by the board's nominating committee. Suppose, for example, that the prior probability that a director nominee is good for the firm is sufficiently high. If the advisor only provides information about the director in the research report but issues *uninformative* public recommendations, then shareholders who do not subscribe to the report will base the decision on their positive priors and predominantly vote in favor of the director. This, however, implies that all non-subscribers tend to vote in the same way, so the aggregate vote outcome is unlikely to be close. Hence, a shareholder who is deciding whether to subscribe to the research report has little incentive to do so, because the probability that his informed vote will matter and sway the outcome towards the value-increasing decision is small.[6]

Suppose, instead, that the advisor issues *informative but biased* recommendations to "create controversy": it always recommends voting against directors who are value-decreasing, but sometimes recommends even against directors who are value-increasing. In this case, non-subscribing shareholders who see a negative recommendation infer that the director could be either good or bad for the firm and are unsure how to vote. This leads to a high chance of a close vote, which, in turn, gives incentives to other shareholders to subscribe to the report so as to vote informatively. In other words, by recommending for the unexpected alternative too often and "creating controversy," the advisor increases the probability that the vote will be close and thus each shareholder's willingness to pay for its research. Of course, the fact that negative recommendations are frequent implies that a positive recommendation is very informative about the director being value-increasing, leading non-subscribing shareholders to vote in favor. Hence, an increase in the probability of a close vote after a negative recommendation comes with a trade-off of a reduction in the probability of a close vote after a positive recommendation. Nevertheless, we show that a recommendation appropriately biased this way is often optimal for the advisor and dominates any other information design, such as issuing on average unbiased but imprecise recommendations, or biasing the recommendations toward the more likely alternative.

At the same time, we show that the advisor has incentives to produce fully informative and unbiased research reports because it helps maximize the revenue from the fees it charges to the subscribing shareholders. In this sense, the interest of the proxy advisor is aligned with those of the shareholders to whom it sells subscriptions. The combination of public recommendations and private research reports is therefore central to the mechanism in our paper: the advisor serves the needs of its clients (subscribers), while limiting and biasing the information it releases through recommendations in order to obtain maximum revenue from

[6] Maug and Rydqvist (2009) and Filali Adib (2020) provide evidence that shareholders are strategic in that they think carefully about the events where their votes can make a difference.

selling the subscriptions.

Our results are consistent with the evidence on how institutional investors use proxy advisory services. According to the survey of institutional investors by Bew and Fields (2012), "virtually unanimously, research participants highlighted the value they derive from ... proxy advisers ... digest[ing] and normaliz[ing] the vast quantities of data present in proxy statements in a short period of time." In other words, institutional investors believe that proxy advisors' research reports do a valuable job in extracting and aggregating information from complex and lengthy proxy statements, consistent with our prediction that the reports will be informative and unbiased. The evidence in Ertimur, Ferri, and Oesch (2013, 2018) supports this view: these papers show that the extent to which shareholders vote against a company's compensation policy and director nominees, respectively, increases with the number and severity of concerns raised by proxy advisors in their research reports. However, while institutions praise the quality of the reports, they rely less on the voting recommendations per se, consistent with our prediction that recommendations are less informative and biased. For example, Bew and Fields (2012) conclude that the "value of ... voting recommendations is distinctly secondary."[7] We discuss this and other empirical evidence in more detail in Section 6.

Proxy advisors are frequently criticized for following a one-size-fits-all approach, i.e., for giving recommendations according to their prescriptive guidelines and without taking into account firm-specific circumstances (e.g., Iliev and Lowry, 2015; Hayne and Vance, 2019). Such a one-size-fits-all approach can be rationalized by our model because it can help the advisor implement recommendations that create controversy. For example, one of proxy advisors' guidelines concerns busy directors: "Generally vote against or withhold from individual directors who sit on more than five public company boards."[8] While a director's busyness is likely to be negatively correlated with his contribution to firm value (e.g., Fich and Shivdasani, 2006), leading to partially informative recommendations, this guideline does not take into account other relevant director characteristics. Consider a nominee who sits on six boards but is an expert in the industry, has many years of experience, and whose other board seats are not too demanding. Whereas the proxy advisor's research report typically

[7] According to one of their respondents, "we don't necessarily agree with everything they say, but they do a tremendous amount of work pulling information together and packaging it, so we can take what we want" (Bew and Fields, 2012, p.13).

[8] See ISS 2021 Voting Guidelines, p. 11. Likewise, Glass Lewis 2021 Voting Guidelines, p. 19–20, specify a similar policy: "we generally recommend that shareholders vote against a director ... who serves on more than five public company boards."

describes all the qualifications of the director and thus allows the subscribers to infer that the director is likely good for the firm, the voting recommendation will be negative, consistent with the "one-size-fits-all" criticism.

We also show that the incentives to create controversy do not arise if the prior probability that the proposal is beneficial is close to 50%. In this case, the advisor designs an informative and unbiased report, but makes its recommendations completely uninformative. Intuitively, with priors close to 50%, an uninformative recommendation will naturally lead to a close vote, increasing shareholders' incentives to subscribe to the report. One way for the advisor to implement such an uninformative recommendation is to always recommend the same action (always vote against or always in favor) on a given type of proposal, without taking into account firm-specific circumstances. For example, both ISS and Glass Lewis 2021 guidelines specify a general recommendation against proposals to classify the board and in favor of proposals to repeal a classified board.

Moreover, our model predicts that deviations from the proxy advisor's recommendations take a specific form: compared to the advisor, shareholders are more predisposed towards the *a priori* more likely alternative, essentially counteracting the bias in recommendations. In the director elections example, suppose that director nominees are *a priori* likely to be value-increasing, so that the advisor biases its recommendations against them. Then, conditional on a positive recommendation, a director must be good for the firm, so both the non-subscribing shareholders observing the positive recommendation and the subscribing shareholders observing the entire report vote in favor. Essentially, all shareholders "rubberstamp" a positive recommendation. In contrast, some directors receiving a negative recommendation are, in fact, value-increasing, leading the subscribers to deviate from the recommendation and vote in their favor. A negative recommendation also generates a lot of uncertainty for non-subscribing shareholders, which leads some of them to vote in favor and others to vote against. Hence, both subscribers and non-subscribers frequently deviate from negative recommendations and vote in favor of the director.

This prediction corresponds to the observed empirical evidence on funds' voting behavior if we assume that management proposals are *a priori* sufficiently likely to be value-increasing.[9] Management proposals that receive a positive recommendation typically pass

[9]Empirically, both proposals to approve director nominees, and management proposals in general, pass with a very high support rate, consistent with the assumption that the priors are high. See Section 6 for more details.

with very high voting support, i.e., are "rubberstamped". In contrast, proposals that receive a negative recommendation often generate a lot of disagreement among shareholders.[10]

These results suggest a natural interpretation of the empirical evidence on funds' voting behavior. Voting in favor of management when proxy advisors recommend "against" is often interpreted as lack of monitoring, passivity, or pro-management bias. In contrast, our model emphasizes that proxy advisors' recommendations may not be the right benchmark since they can be biased against management to create controversy (see also related discussion in Spatt, 2021). Shareholders who support management and deviate from the negative recommendation could be simply correcting the bias in recommendations, rather than voting in a biased way themselves. In this spirit it is striking that large index investors that invest considerable resources in stewardship, such as BlackRock and Vanguard, seem to be more supportive of management than ISS and Glass Lewis (e.g., Bubb and Catan, 2019; Bolton et al., 2020; Brav et al., 2020) and that the votes of smaller mutual fund complexes (who sensibly invest less heavily in stewardship and due diligence) are more closely aligned with ISS (see, e.g., Iliev and Lowry, 2015, and the discussion in Spatt, 2021). Our model predicts that institutional investors that manage larger portfolios are more likely to vote based on their analysis of proxy advisors' reports, rather than purely on the basis of recommendations. Moreover, the votes of such larger shareholders are both informed and unbiased in equilibrium, and in this sense, could be considered a more suitable benchmark than proxy advisory recommendations.

This implication is also important for policy discussions of proxy advisors' biases. A frequently expressed concern is that in addition to selling its research to shareholders, ISS also provides consulting to corporations, which may lead it to bias its recommendations in favor of management in firms that purchase its consulting services (Li, 2018). Our paper emphasizes a very different type of bias, which emerges even if providing voting advice is the only business of the proxy advisor (e.g., as in the case of Glass Lewis). The bias we identify is inherent in selling advice and cannot be alleviated by separating the two businesses or disclosing the advisor's consulting relationships with firms, which has been the focus of policy proposals.

The baseline model assumes that the proxy advisor is the only source of information for the shareholders. In practice, large institutional investors often perform independent research

[10] See, e.g., Table 1 in Ertimur, Ferri, and Oesch (2018) for director elections and Table 1 in Malenko and Shen (2016) for say-on-pay proposals. This evidence is described in more detail in Section 6.

about the issues being voted on (e.g., Iliev and Lowry, 2015; Iliev, Kalodimos, and Lowry, 2021). We therefore consider an extension in which a fraction of the shareholders know the value of the proposal from their own research and cannot learn additional information from the advisor's report. Interestingly, we show that as the fraction of such informed shareholders increases, the advisor's incentives to create controversy become even stronger, in that it designs recommendations that are even more biased and less informative.

Related literature

Our paper contributes to the literature on shareholder voting,[11] including the growing literature on proxy advisory firms. Malenko and Malenko (2019) and Buechel, Mechtenberg, and Wagner (2021) analyze how the presence of proxy advisors affects shareholders' independent research; both papers take the quality of recommendations as given and assume they are unbiased. Levit and Tsoy (2020) show how one-size-fits-all recommendations arise in a cheap talk setting where a biased expert (e.g., a proxy advisor) wants to convince other agents (e.g., shareholders) to accept a certain proposal. Unlike these papers, we focus on the information design problem of an advisor who maximizes its profits from information sale. Ma and Xiong (2021) also study information design by a proxy advisor, but unlike our paper, do not distinguish between a public (recommendation) and private (research report) signal. As a consequence, in their model, the advisor designs biased recommendations only if shareholders themselves are biased; if shareholders maximize firm value, then recommendations are unbiased. In contrast, in our setting, biased recommendations arise even though all shareholders maximize firm value, as a way to increase the probability of a close vote through manipulation of public information. This also distinguishes our paper from Matsusaka and Shu (2021), who study how proxy advisors cater their recommendations to biased shareholders such as SRI funds, and analyze the industry structure that emerges in equilibrium.

In the literature on Bayesian Persuasion, the closest papers to ours examine information design by a biased expert who wants to manipulate the elections to achieve his preferred outcome (Alonso and Camara, 2016; Bardhi and Guo, 2018; Chan et al., 2019). In contrast, in our paper, the designer is unbiased in that he does not get any benefit from the vote going in a particular direction; instead, he maximizes the ex-ante profits from information sale.

[11] E.g., Maug (1999), Brav and Mathews (2011), Bouton et al. (2021), Levit and Malenko (2011), Bar-Isaac and Shapiro (2020), Cvijanovic, Groen-Xu, and Zachariadis (2020), and others.

This implies, in particular, that his information design policies are time-consistent, which is different from most other papers on Bayesian Persuasion. Another feature that distinguishes our paper is that the designer (proxy advisor) designs two signals for two different audiences – one public (for all shareholders) and one private (only for the subscribers). Furthermore, the composition of the latter audience is endogenously determined by the designer's choice. Inostroza (2021) also considers a designer (regulator) designing two signals for multiple audiences, but unlike our paper, these are both public signals on two different dimensions of the bank's fundamentals. In Leitner and Yilmaz (2019), the designer (bank) designs two signals, one of them is observed by the receiver (regulator), and the other is possibly only observed by the designer himself. Goldstein and Huang (2016) and Inostroza and Pavan (2020) study the regulator designing information (stress test) for multiple receivers who have private information, but unlike our paper, assume that the designer sends one signal to all receivers.[12] Chang and Szydlowski (2020) analyze persuasion in a matching market with multiple heterogeneous senders (investment advisors) and multiple heterogeneous receivers (their customers).

Credit rating agencies are another type of information provider to investors. Sangiorgi and Spatt (2017) provide an overview of the literature on credit rating agencies and discuss their similarities and differences from proxy advisors. In the context of our paper, the most important differences between the two are 1) the pricing models – whereas credit rating agencies are paid by the issuers, proxy advisors are paid by investors; and 2) the nature of externalities between the users of information – whereas traders in financial markets compete for profits, shareholders' objectives in voting are often aligned at value maximization. There are also certain similarities, such as the issue of multiple (albeit different from the proxy advisory setting) audiences explored in Frenkel (2015) and Bouvard and Levy (2018), and the provision of both paid and unpaid signals explored in Fulghieri, Strobl, and Xia (2014).

Finally, our paper is related to the literature on the sale of information to traders in financial markets (e.g., Admati and Pfleiderer, 1986, 1990; Fishman and Hagerty, 1995; Cespa, 2008; Garcia and Sangiorgi, 2011). One important conclusion in this literature is that the seller may benefit from adding noise to the information it sells, as a way to decrease the leakage of information through prices. Differently from this result, we show that the seller of information to voters benefits from selling the most precise information to those subscribing to its report. Instead, to increase the value of this information, it strategically

[12] Other papers studying information design in the context of stress tests include Goldstein and Leitner (2018), Leitner and Williams (2020), and Orlov, Zryumov, and Skrzypacz (2020).

biases the public information it reveals. Thus, both the result and the underlying mechanism are different from this literature.

2 Model setup

The firm is owned by $N \geq 3$ shareholders, where N is odd. Each shareholder owns one share in the firm and has one vote. There is a proposal to be voted at the shareholder meeting, which would be approved if at least $\frac{N+1}{2}$ shareholders vote for it. Let $d \in \{0, 1\}$ denote the decision on the proposal, with $d = 1$ ($d = 0$) referring to the proposal approval (rejection).

The value of the proposal to shareholder i, $u_i(d, \theta)$, depends on the unknown state $\theta \in \{0, 1\}$ and on the importance of the proposal to the shareholder, v_i, as follows:

$$u_i(d, \theta) = v_i \cdot u(d, \theta),$$

where

$$\begin{aligned} u(1, \theta) &= \begin{cases} 1, & \text{if } \theta = 1, \\ -1, & \text{if } \theta = 0, \end{cases} \\ u(0, \theta) &= 0. \end{aligned}$$

In other words, approving the proposal increases (decreases) shareholder value if $\theta = 1$ ($\theta = 0$), while rejecting the proposal and maintaining the status quo leaves firm value unchanged. The ex-ante probability that the proposal is value-increasing is $\Pr(\theta = 1) = \mu \in (0, 1)$.

Thus, all shareholders' interests are perfectly aligned, but the extent to which they care about the proposal, v_i, may differ across them. There are multiple reasons for this heterogeneity in practice. First, v_i is likely to depend on the sensitivity of the fund manager's compensation to the value of its portfolio firm, which differs significantly across funds. For example, holding ownership constant, v_i is likely to be highest for hedge funds, lowest for index funds, and medium for actively-managed mutual funds.[13] Second, heterogeneity in v_i can be due to the fact that the voting practices of shareholders are scrutinized by regulators and investors to a different extent – for example, retail investors may have a lower v_i than

[13] See Corum, Malenko, and Malenko (2021) for a model in which active and passive institutional investors have different equilibrium sensitivities v_i to the value of their portfolio firms due to different fees they charge to investors.

institutional investors for this reason. In addition, v_i can indirectly capture the shareholder's position in the firm, although this interpretation needs to be used cautiously given our assumption of equal stakes across shareholders. Lewellen and Lewellen (2021) provide empirical estimates of v_i across institutional investors and firms. Value v_i is an independent (across shareholders) draw from a distribution with c.d.f. $H(\cdot)$ over $[\underline{v}, \overline{v}]$ with $0 \leq \underline{v} < \overline{v} \leq \infty$. For example, in the special case where $H(\cdot)$ is degenerate at v, shareholders are homogeneous; and in the special case where $H(\cdot)$ has two atoms, v_L and v_H, we can think of two groups of shareholders – those that care little about voting and those that care more substantially. Each shareholder i knows her own preference parameter v_i, but not the preference parameters of other shareholders, aside from distribution $H(\cdot)$. The role of heterogeneous v_i is to produce variation across shareholders in their incentives to pay for advice so as to make informed voting decisions.

Each shareholder is initially uninformed about the state. There is a seller of information, the proxy advisor, that has an informative signal about the state. For simplicity, we assume that the proxy advisor knows the state with certainty. The advisor prepares two signals, a private signal available only to the subscribers and a public signal available to everyone. The private signal, denoted $\mathcal{R} = \left(R, \{\phi(\cdot|\theta)\}_{\theta \in \{0,1\}}\right)$, consists of a finite signal space R and two distributions $\{\phi(\cdot|\theta)\}_{\theta \in \{0,1\}}$ of signal realizations r over R, one for each state $\theta \in \{0, 1\}$. The public signal, denoted $\mathcal{S} = \left(S, \{\gamma(\cdot|r)\}_{r \in R}\right)$, consists of a finite signal space S and a family of distributions $\{\gamma(\cdot|r)\}_{r \in R}$ of signal realizations s over S for each private signal realization $r \in R$. Let $\tau(s)$ denote the probability of observing public signal $s \in S$ implied by information policy $\mathcal{S}$:

$$\tau(s) \equiv \sum_{r \in R, s \in S} \gamma(s|r) \left(\phi(r|1)\mu + \phi(r|0)(1-\mu)\right).$$

We will refer to the private signal policy $\mathcal{R}$ with signal realizations $r \in R$ as the *research report* of the proxy advisor and to the public signal policy $\mathcal{S}$ with signal realizations $s \in S$ as the *voting recommendation* of the proxy advisor. This formulation means that the research report is informative about the true state θ, while the voting recommendation determines how the content of the research report is mapped into a voting recommendation. As discussed in the introduction, the assumption that the recommendation is observed by all investors, while the research report is only learned by the proxy advisor's subscribers, corresponds to reality: for important decisions, such as proxy contests or contentious say-on-pay votes, proxy advisors' recommendations are typically publicly revealed in the media, but their research reports, which contain a comprehensive analysis of the proposal, are only available to the advisors' institutional clients. One way to think of policy $\mathcal{S}$ is that it represents the voting guidelines that each proxy advisor publicly discloses every year: for each type of proposal, these guidelines describe how the proxy advisor will issue its recommendation on this proposal in a given company based on various aspects of the company's corporate governance and performance.

Note also that while our baseline model analyzes subscriptions to only one proposal and in one firm, Section 6 discusses how our analysis and results can be extended to a setting where the subscription covers multiple proposals and firms.

Following the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011, and follow-up work), we assume that the advisor commits to policies $\mathcal{R}$ and $\mathcal{S}$. As we discuss later, the seller's ex-ante optimal information policy turns out to be dynamically consistent for the seller in our model, unlike in Kamenica and Gentzkow (2011) and most other models of Bayesian persuasion.

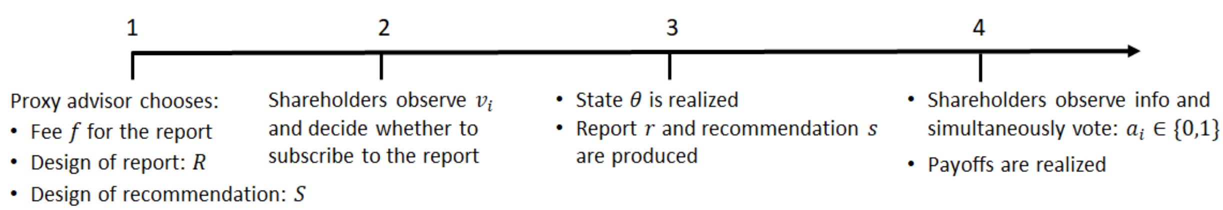


Figure 1. Timeline of the model.

The timeline of the model is shown in Figure 1. At stage 1, the advisor chooses the information policy $(\mathcal{R}, \mathcal{S})$ and fee f that it charges to shareholders for subscribing to its research report $\mathcal{R}$. At stage 2, having observed the information policy $(\mathcal{R}, \mathcal{S})$, fee f, and his realization of proposal importance v_i, each shareholder i simultaneously and non-cooperatively decides on whether to pay fee f to subscribe to the advisor's report or not. At stage 3, the advisor observes θ and issues a research report $r \in R$ and recommendation $s \in S$, so that all shareholders observe the realization of s and shareholders that subscribed to the advisor also see the research report r. At stage 4, each shareholder decides whether to vote "for" $(a_i = 1)$ or "against" $(a_i = 0)$ the proposal, and the proposal gets implemented if it is approved by the majority.

Shareholders maximize $u_i(d,\theta)$ minus any costs of information acquisition, and the proxy advisor maximizes its expected profits from selling the subscriptions. The equilibrium concept is a symmetric Bayes-Nash equilibrium.

We conjecture and later verify that it is optimal for the proxy advisor to design a fully informative research report, i.e., $R = \{0,1\}$ and $r = \theta$. Thus, by subscribing to the proxy advisor's services, a shareholder learns the state with certainty. Given this, the proxy advisor's problem is how to design the public recommendation, given each possible realization of state θ: $\mathcal{S} = \left(S, \{\gamma(\cdot|\theta)\}_{\theta\in\{0,1\}}\right)$. For example, in the case of a binary recommendation space $S = \{0,1\}$, which we will show to be optimal, the recommendation policy is characterized by two probabilities, $\Pr(s=1|\theta=1) = \gamma(1|1)$ and $\Pr(s=1|\theta=0) = \gamma(1|0)$.

3 Solution of the model

We solve the model by backward induction. We focus on the case in which the research report $\mathcal{R}$ is fully informative, and solve for the equilibrium in the voting game, the equilibrium subscription decisions, pricing of information, and the public recommendation design. In Section 3.5, we complete the solution by proving that making the research report truthful is optimal for the seller.

3.1 Voting stage

Since the payoff of a shareholder is proportional to the importance v_i of the proposal to him, his vote does not depend on v_i, and depends only on his information set (s or $\{s,r\}$), as well as the information that he infers from the fact that his vote is pivotal. Specifically, as in the literature on strategic voting (e.g., Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998), shareholders take into account that their vote only matters in situations where it changes the voting outcome and thus rationally condition their voting decisions on the information that is true in this situation.[14]

A shareholder finds it optimal to vote "for" ($a_i = 1$) if his posterior probability of $\theta = 1$, given his information set and what he infers from the fact that his vote is pivotal, exceeds

[14]Such strategic voting is related to the idea of the "winner's curse:" both in auctions and in voting, an agent's action only matters in a particular situation – when his bid is the highest and when his vote is pivotal, respectively. Since other agents (bidders and voters, respectively) have valuable information, the rational agent conditions his decision on the information that must be true when his decision matters.

$\frac{1}{2}$. If this posterior is below $\frac{1}{2}$, he finds it optimal to vote "against" ($a_i = 0$). If it is exactly equal to $\frac{1}{2}$, the shareholder is indifferent.

First, consider a shareholder who subscribed to the research report. Since he knows the state with certainty from the research report, it is optimal for his vote to match the state: $a_i = \theta$.

Second, consider a shareholder who did not subscribe to the research report. To solve for his optimal voting strategy, we need to calculate $\Pr(\theta = 1|s, Piv)$, where s is the realization of the public signal generated by the proxy advisor (the voting recommendation) and Piv is the event that this shareholder's vote is pivotal for the vote outcome. Let $\mu_s = \Pr(\theta = 1|s)$ denote the posterior probability of $\theta = 1$ implied by signal realization s, and let q denote the probability, as perceived by shareholder i, with which each other shareholder is a subscriber of the proxy advisor's research report. If every non-subscribing shareholder votes for the proposal with probability π, then using Bayes' rule,

$$\Pr(\theta = 1|q, \mu_s, Piv) = \frac{\Pr(Piv|\theta = 1, q, \mu_s)\,\mu_s}{\Pr(Piv|\theta = 1, q, \mu_s)\,\mu_s + \Pr(Piv|\theta = 0, q, \mu_s)\,(1 - \mu_s)}. \tag{1}$$

Intuitively, a shareholder constructs a posterior of state $\theta = 1$ having two pieces of information: the recommendation of the proxy advisor and the additional information he learns from the fact that he is pivotal. The first signal corresponds to terms μ_s and $1 - \mu_s$ in (1), while the second signal corresponds to terms $\Pr(Piv|\theta = 1, q, \mu_s)$ and $\Pr(Piv|\theta = 0, q, \mu_s)$ in (1).

If $\Pr(\theta = 1|q, \mu_s, Piv) \geq \frac{1}{2}$ for $\pi = 1$, there is an equilibrium in which all non-subscribing shareholders vote for the proposal. If $\Pr(\theta = 1|q, \mu_s, Piv) \leq \frac{1}{2}$ for $\pi = 0$, there is an equilibrium in which all non-subscribing shareholders vote against. Finally, if $\Pr(\theta = 1|q, \mu_s, Piv) = \frac{1}{2}$ for some $\pi \in (0, 1)$, there is an equilibrium in which each non-subscribing shareholder randomizes between voting for and against with probabilities π and $1 - \pi$, respectively. The next proposition characterizes the equilibrium in the voting game for any recommendation realization s:

Proposition 1. *Consider the voting game that follows a recommendation realization s. The equilibrium takes the following form. If a shareholder is a subscriber, he votes $a_i = \theta$.*

If a shareholder is not a subscriber and $\mu_s \in (0,1)$*, he votes* $a_i = 1$ *with probability*

$$\pi(q,\mu_s) = \frac{z_s(1-2q) - 1 + \sqrt{(z_s-1)^2 + 4q^2 z_s}}{2(z_s-1)(1-q)}, \tag{2}$$

where $z_s \equiv \left(\frac{\mu_s}{1-\mu_s}\right)^{\frac{2}{N-1}}$*. If a shareholder is not a subscriber and* $\mu_s = 1$ *(*$\mu_s = 0$*), he votes* $a_i = 1$ *with probability one (zero).*

This equilibrium is intuitive. All shareholders with precise information (subscribers) vote according to their information. In contrast, shareholders with imprecise information (non-subscribers) randomize between voting for and against the proposal, with the property that the expected fraction of votes for the proposal obtained from these shareholders is increasing in the belief μ_s that the proposal is value-improving.

Figure 2 illustrates Proposition 1 and highlights the property that the sensitivity of votes of non-subscribers to their posterior μ_s is affected by the expected fraction of subscribers, q. Intuitively, a shareholder knows that his vote matters only in the event that the votes of other shareholders are split and tries to infer information from that event. Since he expects the non-subscribers to likely vote along the posterior belief μ_s, the fact that the vote is split implies that the subscribers were more likely to vote against belief μ_s than initially expected. Thus, the shareholder updates the probability that the state is $\theta = 1$, and his vote becomes less reliant on μ_s. The extent of this learning depends on q. If q is very low, the shareholder learns very little from the fact that the vote is split, since almost no vote is perfectly informed. As a consequence, the voting strategy relies heavily on whether posterior μ_s is above $\frac{1}{2}$ or below $\frac{1}{2}$, as illustrated by the blue line in Figure 2 for $q = 0.01$. In the limit when $q \to 0$, $\pi(q,\mu_s)$ converges to a step integer function $1\left\{\mu_s > \frac{1}{2}\right\}$. In contrast, if q is relatively high, a non-subscriber learns quite a bit from the fact that the votes of others are split, since the probability that each vote is perfectly informed is now higher. As a consequence, in equilibrium, the probability that he votes "for," $\pi(q,\mu_s)$, becomes less sensitive to posterior μ_s around $\mu_s = \frac{1}{2}$. This is illustrated by the yellow and orange lines in Figure 2 for $q = 0.2$ and $q = 0.45$, respectively.

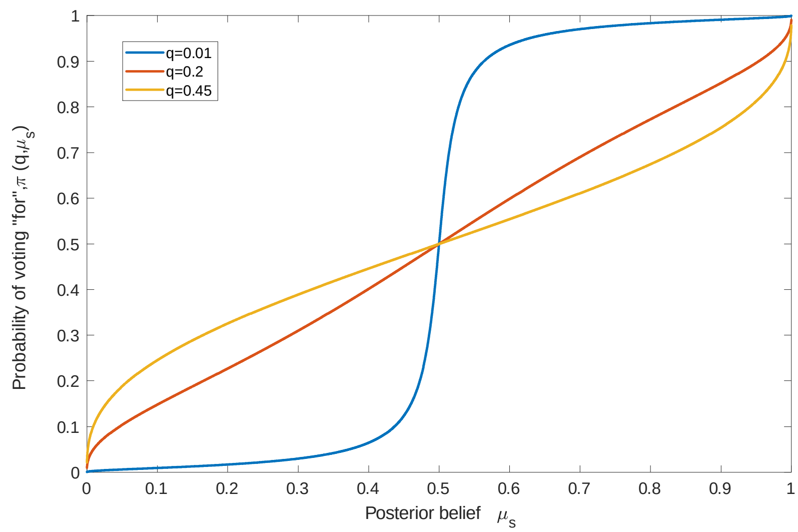


Figure 2. Illustration of Proposition 1. This figure shows the probability that a non-subscribing shareholder votes for the proposal as a function of his posterior μ_s for three different values of q. The parameters are $N = 7$, $q \in \{0.01, 0.2, 0.45\}$.

3.2 Information acquisition stage

While all shareholders can utilize the recommendation of the proxy advisor, the incentive to purchase the research report reflects the incremental value from the shareholder's perspective of an improved outcome due to the value to that shareholder of an enhanced voting decision.[15]

Given the equilibrium in the voting game from Proposition 1, we can calculate the private value to an individual shareholder from the research report, i.e., from learning the state. It will be a function of probability q, with which he expects each other shareholder to subscribe to the report and therefore vote informatively, the recommendation policy $\mathcal{S}$, and type v_i of the shareholder. Let $V_i(q, \mathcal{S})$ denote this value.

To obtain $V_i(q, \mathcal{S})$, consider shareholder i's value from learning the state for a particular realization $s \in S$ that induces posterior μ_s. If $\mu_s = 1$ or $\mu_s = 0$, then the shareholder already knows the state with certainty from observing the public recommendation s. In addition, his

[15] A traditional challenge confronting information producers, such as proxy advisory firms, is the free-rider problem which leads to difficulty in being paid for the information that they generate (e.g., Arrow (1962)), but the presence of the research report that is not publicly available facilitates the ability of the proxy advisory firm to be paid for the information that it generates. The presence of the proxy-advisory firm helps overcome the free-rider problem among shareholders, provided that there is not a free-rider problem that would prevent the intermediary from being paid for the information it produces.

vote is never pivotal in this case, since all shareholders vote the same way. Thus, if $\mu_s = 1$ or $\mu_s = 0$, the shareholder's value from the research report is zero. If $\mu_s \in (0,1)$, then shareholder i's value from learning the state is positive. His vote changes the decision of the firm with probability $\Pr(Piv|q,\mu_s)$, and, conditional on his vote being pivotal, learning the state changes the probability of the correct decision ($d=\theta$) from $\frac{1}{2}$ to 1.[16] Using this and the fact that the importance of the proposal to shareholder i is v_i, his value from learning the state is $\frac{1}{2}v_i \Pr(Piv|q,\mu_s)$. Aggregating over all possible realizations of the recommendation $s \in S$, we obtain that the value of the research report to shareholder i is

$$V_i(q,\mathcal{S}) = v_i \cdot V(q,\mathcal{S}), \tag{3}$$

where

$$V(q,\mathcal{S}) = \frac{1}{2}\sum_{s\in S} \Pr(Piv|q,\mu_s)\,\tau(s) \tag{4}$$

and $\tau(s) = \mu\gamma(s|1) + (1-\mu)\gamma(s|0)$ is the probability of observing recommendation s. Intuitively, $V(q,\mathcal{S})$ is the average (before the recommendation $s \in S$ is realized) probability that each shareholder is pivotal multiplied by the benefit $\frac{1}{2}$ of learning the state conditional on being pivotal.

Hence, shareholder i buys the report if and only if $v_i V(q,\mathcal{S}) \geq f$, or equivalently,

$$v_i \geq \hat{v} \equiv \frac{f}{V(q,\mathcal{S})}.$$

It follows that, given fee f, the fraction of shareholders subscribing to the advisor's report is $1 - H\left(\frac{f}{V(q,\mathcal{S})}\right)$. We can equivalently rewrite this expression as an inverse demand function. Specifically, to ensure that, on average, fraction q of shareholders subscribe to the proxy advisor's report, the fee must be:

$$f = V(q,\mathcal{S})\,H^{-1}(1-q) = V(q,\mathcal{S})\,\tilde{h}(1-q), \tag{5}$$

where $\tilde{h}(\cdot) \equiv H^{-1}(\cdot)$ is an increasing function.

We can summarize these arguments in the following proposition:

[16] The probability of voting correctly ($v_i = \theta$), conditional on signal s and shareholder i being pivotal, is $\frac{1}{2}$, because conditional on this information, both states are equally likely by Proposition 1.

Proposition 2. *For a given fee f and public recommendation policy $\mathcal{S}$, a shareholder subscribes to the proxy advisor's report if and only if the importance of the proposal to him exceeds $f/V(q,\mathcal{S})$. The expected fraction q of shareholders that subscribe to the report is given by the solution to (5), if $q \in (0,1)$.*

3.3 Seller's public recommendation design

The seller's problem is to choose fee f and the information policy $\mathcal{S}$ to maximize expected profits Nqf, where f is given by (5). Equivalently, we can rewrite the seller's problem as choosing q and $\mathcal{S}$ to maximize

$$\max_{q,S} q\tilde{h}(1-q)\left(\sum_{s\in S}\Pr(Piv|q,\mu_s)\,\tau(s)\right) \tag{6}$$

subject to the constraint that the information policy is Bayes plausible, i.e., that the expected posterior probability of each state equals its prior probability (this requirement is necessary to ensure Bayesian rationality):

$$\sum_{s\in S}\mu_s\tau(s) = \mu. \tag{7}$$

For a fixed q, problem (6)–(7) can be solved using the concavification approach from Kamenica and Gentzkow (2011). Let $P(\mu|q)$ be the concave closure of $\Pr(Piv|q,\mu)$. Then, $P(\mu|q)$ is the largest payoff $\sum_{s\in S}\Pr(Piv|q,\mu_s)\,\tau(s)$ the seller can achieve when the prior is μ and the probability with which each shareholder subscribes to the seller's research report is q. The next lemma establishes the key properties of $\Pr(Piv|q,\mu)$, and Figure 3 illustrates them graphically.

Lemma 1. *If $q<\frac{1}{2}$, there exist $\mu_l \in \left(0,\frac{1}{2}\right)$, $\mu_h \in \left(\frac{1}{2},1\right)$, and $\varepsilon>0$, such that $\Pr(Piv|q,\mu)$ is strictly convex in μ at $\mu \in (0,\mu_l)$ and $\mu \in (\mu_h,1)$, and strictly concave in μ at $\mu \in \left(\frac{1}{2}-\varepsilon,\frac{1}{2}+\varepsilon\right)$. If $q>\frac{1}{2}$, $\Pr(Piv|q,\mu)$ is strictly concave in μ if μ is sufficiently close to $\frac{1}{2}$, and if μ is close to 0 and 1.*

Given Lemma 1, it is straightforward to characterize the optimal recommendation policy when μ is close to 1, 0, and $\frac{1}{2}$. According to Lemma 1, if $q<\frac{1}{2}$, $P(\mu|q)$ is linear at μ sufficiently close to 0 and 1 and strictly concave at μ close to $\frac{1}{2}$. It follows that the optimal

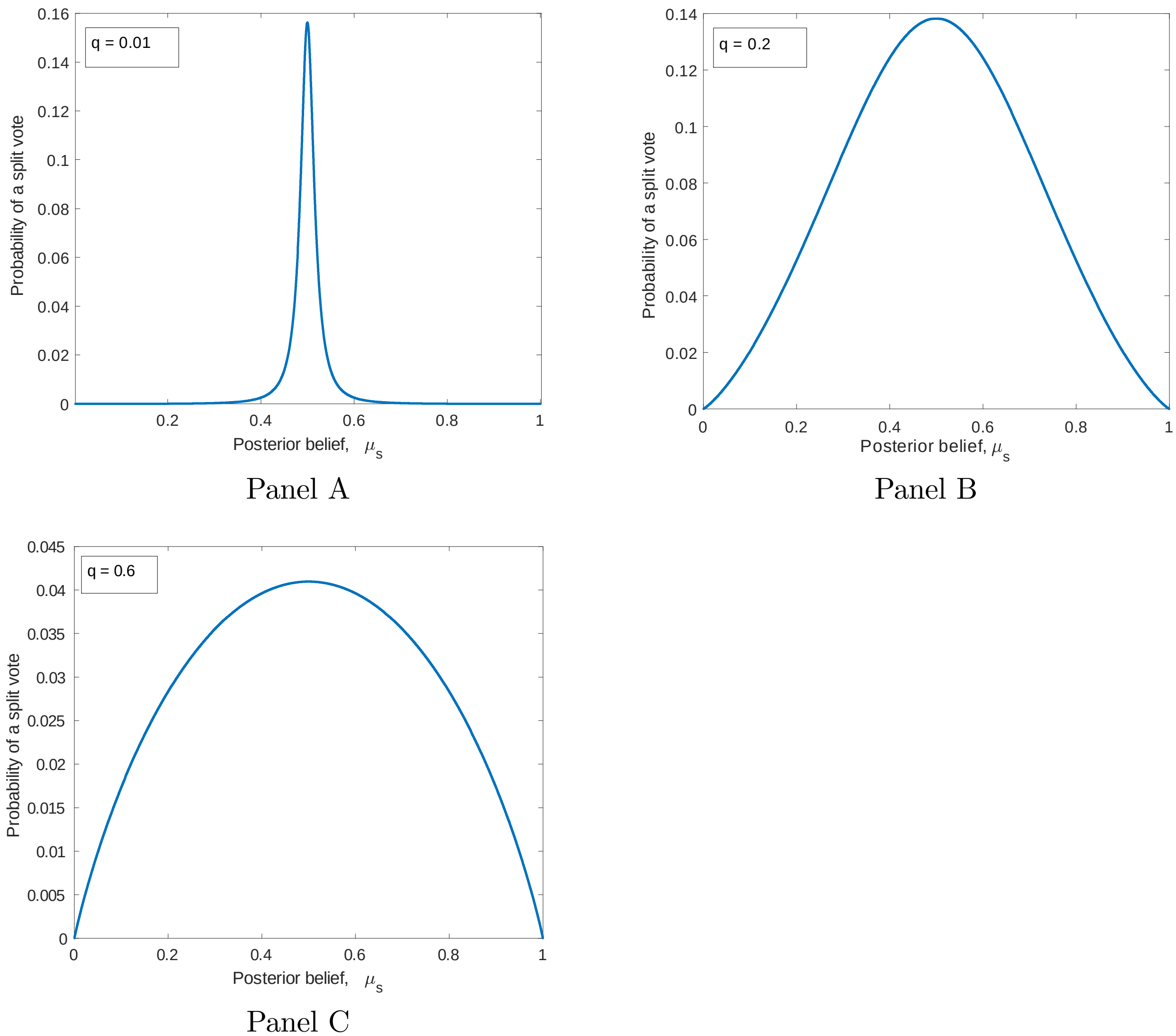


Figure 3. Illustration of Lemma 1 for three different values of q. Panels A and B show cases of $q < \frac{1}{2}$, when $\Pr(Piv|q, \mu)$ is convex for low and high values of μ but concave for middle values of μ. Panel C shows the case of $q > \frac{1}{2}$, when $\Pr(Piv|q, \mu)$ is concave for all values of q. The parameters are $N = 7$, $q = 0.01$ (Panel A), $q = 0.2$ (Panel B), $q = 0.6$ (Panel C).

recommendation policy is characterized by the following proposition, illustrated in Figure 4.

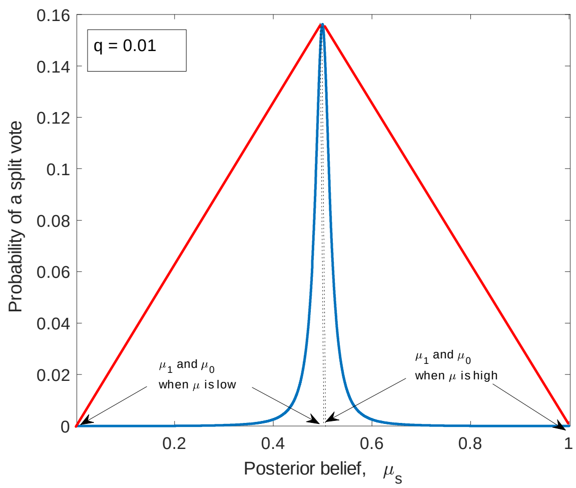
Proposition 3 (optimal recommendation design for a fixed fraction of subscribers). *The optimal public recommendation is binary, $S = \{0,1\}$, and has the following properties:*

1. *If $q < \frac{1}{2}$ and $\mu < \mu_l$, signal $s = 1$ induces a belief $\mu_1 \in \left(\mu, \frac{1}{2}\right)$, satisfying $\frac{\Pr(Piv|q,\mu_1)}{\mu_1} = \frac{\partial}{\partial \mu} \Pr\left(Piv|q,\mu_1\right)$, and signal $s = 0$ induces a belief $\mu_0 = 0$.*
2. *If $q < \frac{1}{2}$ and $\mu > \mu_h$, signal $s = 1$ induces a belief $\mu_1 = 1$, and signal $s = 0$ induces a belief $\mu_0 \in \left(\frac{1}{2}, \mu\right)$, satisfying $\frac{\Pr(Piv|q,\mu_0)}{1-\mu_0} = -\frac{\partial}{\partial \mu} \Pr\left(Piv|q,\mu_0\right)$.*
3. *If $q < \frac{1}{2}$ and $\mu \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right)$, or if $q \geq \frac{1}{2}$, the optimal signal is uninformative.*

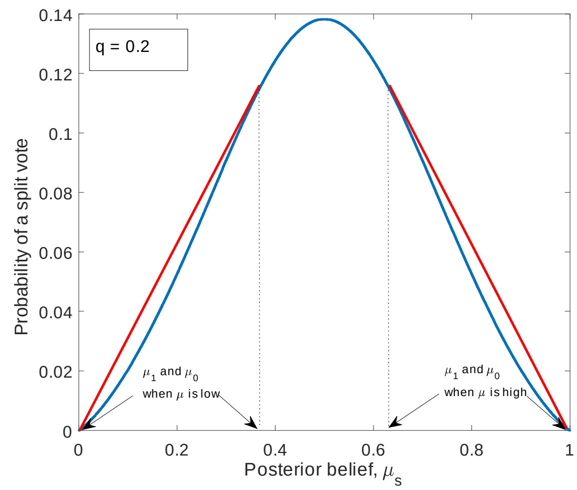
If the prior beliefs are sufficiently asymmetric and the proxy advisor does not have too many subscribers, the optimal signal creates controversy. For example, consider $\mu > \mu_h$ (the case $\mu < \mu_l$ is analogous). Proposition 3 shows that the *positive* recommendation reveals that the state is 1 and leads shareholders to rubberstamp the proposal: all shareholders vote in favor, regardless of whether they are subscribers or non-subscribers, so the research report is irrelevant.

However, the *negative* recommendation only reveals that the probability of $\theta = 1$ is close to 50%. The seller achieves this by issuing a negative recommendation in all cases where $\theta = 0$, but also in many cases where $\theta = 1$. For example, as discussed in the introduction, such information design can be implemented by proxy voting guidelines that commit to give a negative recommendation if a certain condition is satisfied (e.g., a director has too many board seats). From the Bayes plausibility constraint (7), the probability of a negative signal is $\Pr\left(s = 0\right) = \frac{1-\mu}{1-\mu_0}$, which exceeds the probability that the correct decision is to reject the proposal ($\Pr\left(\theta = 0\right) = 1 - \mu$) by a factor of $\frac{1}{1-\mu_0}$. Intuitively, by making negative recommendations frequent but with a confusing meaning, the advisor increases the probability of a close vote upon a negative recommendation and thereby each shareholder's willingness to pay for advice.

Proposition 3 also shows that it is suboptimal to give any information for free if either the prior is close to 50% or if sufficiently many shareholders are subscribers. The former result is intuitive: If there is already a lot of uncertainty about the correct decision, then the probability of a close vote is high, so shareholders have incentives to become informed, and there is no benefit to manipulating public information. Regarding the latter result, as Figure 2 shows, the impact of a marginal reduction in the belief $\mu > \frac{1}{2}$ on the probability of voting in favor is greater when q is higher. Thus, the benefit from inducing beliefs close to 50% is smaller when q is higher. Since there is also a cost, uninformative recommendations become optimal if q is high enough.



Panel A



Panel B

Figure 4. Optimal public recommendation for different values of q. The left panel illustrates the case of q=0.01. In this case, if μ is low (below 0.4997), the optimal public recommendation induces posteriors 0 and 0.4997; if μ is high (above 0.5003), the optimal public recommendation induces posteriors 0.5003 and 1. The right panel illustrates the case of q=0.2. In this case, if μ is low (below 0.3721), the optimal public recommendation induces posteriors 0 and 0.3721; if μ is high (above 0.6279), the optimal public recommendation induces posteriors 0.6279 and 1. If μ is between the two cut-offs, the optimal public recommendation is uninformative.

3.4 Pricing of information by the seller

Next, consider the optimal pricing of information by the seller. Suppose the seller chooses a fee such that it sells, in expectation, to a fraction q of shareholders. Then her expected profits are $NqP(\mu, q)\,\tilde{h}(1-q)$, where, as discussed above, $P(\mu, q)$ is the highest probability of a shareholder being pivotal that the seller can achieve when the prior is μ and the probability that shareholders subscribe to the report is q. Thus, the optimal fee induces q that solves

$$\max_{q} qP(\mu, q)\,\tilde{h}(1-q)\,. \tag{8}$$

Consider the case in which the prior is sufficiently asymmetric, $\mu > \mu_h$ or $\mu < \mu_l$ from Proposition 3. According to Proposition 3, the optimal recommendation is biased if $q < \frac{1}{2}$ and uninformative if $q \geq \frac{1}{2}$. Thus, there are two possibilities to consider.

The first possibility is that the solution to (8) is $q < \frac{1}{2}$. In other words, the seller sets a high enough fee so that in expectation fewer than 50% of shareholders become subscribers, and designs public recommendations that are biased against the status quo. What will be the profit-maximizing choice of q in this range? First, consider the case of $\mu < \mu_l$. Since the optimal recommendation induces posteriors μ_1 and 0 with probabilities $\frac{\mu}{\mu_1}$ and $1-\frac{\mu}{\mu_1}$, respectively, (8) is equivalent to

$$\mu \max_{q} q\tilde{h}(1-q)\,\frac{\Pr(Piv|q,\mu_1)}{\mu_1} = \mu \max_{q,m} q\tilde{h}(1-q)\,\frac{\Pr(Piv|q,m)}{m}, \tag{9}$$

where the equality follows from the fact that, by Proposition 3, μ_1 maximizes $\frac{\Pr(Piv|q,m)}{m}$ over m. Using the expression for $\Pr(Piv|q,m)$, (25), (8) implies

$$\max_{q,m} q\tilde{h}(1-q)\left((q+(1-q)\,\pi(q,m))\,(1-q)\,(1-\pi(q,m))\right)^{\frac{N-1}{2}}. \tag{10}$$

We note two properties of (10). First, the optimization problem does not depend on the prior μ. Thus, as long as the prior is sufficiently asymmetric to make biased recommendations optimal, it is optimal to set the fee to induce the same (independent of the prior) fraction of subscribers q and the same posteriors that follow public recommendations. Second, the optimization problem depends on m only via m affecting the votes of non-subscribing shareholders. It follows that (10) is maximized when

$$q+(1-q)\,\pi(q,\mu_1) = \frac{1}{2}, \tag{11}$$

i.e., posterior μ_1 is such that the probability of voting for the proposal, conditional on it being good and the recommendation being positive, is exactly 50%. This, in turn, implies that the optimal posterior that the positive recommendation induces is less than 50% and given by

$$\mu_1 = \frac{\left(1-4q^2\right)^{\frac{N-1}{2}}}{1+\left(1-4q^2\right)^{\frac{N-1}{2}}}. \tag{12}$$

Using similar arguments for $\mu > \mu_h$, the optimal posterior that the negative recommendation induces is greater than 50% and given by

$$\mu_0 = 1 - \frac{\left(1-4q^2\right)^{\frac{N-1}{2}}}{1+\left(1-4q^2\right)^{\frac{N-1}{2}}} = \frac{1}{1+\left(1-4q^2\right)^{\frac{N-1}{2}}} \tag{13}$$

Then, from (10) it follows that the optimal q is

$$q^* = \arg\max_q q\tilde{h}\left(1-q\right). \tag{14}$$

To sum up, if the seller chooses to design a biased recommendation, it is optimal to set the fee to sell research to a fraction q^* of the shareholders and to design a recommendation that induces posterior 0 and μ_1 (if μ is low) or μ_0 and 1 (if μ is high).

The second possibility is that the solution to (8) is $q \geq \frac{1}{2}$. In this case, the optimal q solves

$$\max_q q\tilde{h}\left(1-q\right)\left(\left(q+\left(1-q\right)\pi\left(q,\mu\right)\right)\left(1-q\right)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}}. \tag{15}$$

Note that (15) is weakly below (10) since it has one fewer degree of flexibility: the posterior belief of non-subscribing shareholders is fixed at μ rather than being a parameter of choice for the seller. Therefore, if $q^* < \frac{1}{2}$, then it is optimal for the seller to design a biased recommendation and sell to a fraction q^* of the shareholders. By contrast, if $q^* \geq \frac{1}{2}$, then the optimal recommendation is uninformative and the fee is such that the expected fraction of subscribers is

$$q^{**} = \arg\max_q q\tilde{h}\left(1-q\right)\left(\left(q+\left(1-q\right)\pi\left(q,\mu\right)\right)\left(1-q\right)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}}. \tag{16}$$

This analysis is summarized in the following proposition:

Proposition 4 (optimal pricing and design of information). *Suppose that prior μ is sufficiently asymmetric. The optimal public recommendation and the price of a research report are as follows.*

1. *If q^*, defined by (14), is below $\frac{1}{2}$ and μ is low, then recommendation $s = 1$ occurs with probability $\frac{\mu}{\mu_1}$ and induces belief $\mu_1 = \frac{\left(1-4q^{*2}\right)^{\frac{N-1}{2}}}{1+\left(1-4q^{*2}\right)^{\frac{N-1}{2}}}$; and recommendation $s = 0$ occurs with probability $\frac{\mu_1-\mu}{\mu_1}$ and induces belief $\mu_0 = 0$. The price of the research report is $f = 2^{-N}\mu\tilde{h}\left(1-q^*\right)$, and the probability that a shareholder subscribes to it is q^*.*

2. *If q^* is below $\frac{1}{2}$ and μ is high, then recommendation $s = 1$ occurs with probability $\frac{\mu-\mu_0}{1-\mu_0}$ and induces belief $\mu_1 = 1$; and recommendation $s = 0$ occurs with probability $\frac{1-\mu}{1-\mu_0}$ and induces belief $\mu_0 = \frac{1}{1+\left(1-4q^{*2}\right)^{\frac{N-1}{2}}}$. The price of the research report is $f = 2^{-N}\left(1-\mu\right)\tilde{h}\left(1-q^*\right)$, and the probability that a shareholder subscribes to it is q^*.*

3. *If q^* is above $\frac{1}{2}$, then the recommendation is uninformative, the price of the research report is $f = \frac{1}{2}q^{**}\Pr\left(Piv|q^{**},\mu\right)\tilde{h}\left(1-q^{**}\right)$, where q^{**} is given by (16), and the probability that a shareholder subscribes to it is q^{**}.*

To illustrate the intuition for Proposition 4, consider an example in which the intensity of shareholders' preferences about the proposal is distributed according to the power distribution with parameter α:

Example: $H\left(x\right) = x^{\alpha}$. In this case, $\tilde{h}\left(1-q\right) = \left(1-q\right)^{\frac{1}{\alpha}}$, so $q^* = \frac{\alpha}{\alpha+1}$. Therefore, if $\alpha < 1$, then a profit-maximizing seller finds it optimal to design a biased recommendation and sell research report to fraction $\frac{\alpha}{\alpha+1}$ of shareholders (in expectation). In contrast, if $\alpha > 1$, then a profit-maximizing seller finds it optimal to design an uninformative recommendation and sell research report to fraction q^{**} of shareholders.

Intuitively, $\alpha < 1$ means that the distribution of the intensity of shareholders' preferences has a positive skew: There are many shareholders who care little about the proposal, and relatively few who care quite a bit. In this case, it is optimal to sell the research report to a relatively small fraction of shareholders who care significantly about the proposal, and to increase the private value of the research report by inducing controversy via a biased public recommendation. In contrast, $\alpha > 1$ means that the distribution of the intensity of shareholders' preferences has a negative skew: There are many shareholders who care a lot about the proposal and some who care very little. In this case, it is optimal to sell the research report to a large fraction of shareholders. Since there are few shareholders who do not have access to the research report, it is optimal to have an uninformative public recommendation so as not to dilute the value of information available in the research report.

3.5 Seller's private research report design

So far, we have solved for the optimal public recommendation design, the fee charged for the research report, and the equilibrium in the voting game conjecturing that the seller finds it optimal to design the private research report that reveals the state truthfully. The next proposition verifies that this conjecture is indeed correct:

Proposition 5 (optimality of a fully informative research report). *A fully informative private research report, i.e., the one with $R = \{0, 1\}$ and $r = \theta$, is optimal for the seller.*

Loosely speaking, a fully informative private research report is optimal, because adding noise to a private research report only dilutes its value and thus lowers the willingness to pay of shareholders who subscribe to it. The more formal intuition is a combination of two steps. First, observe that for any research report with an arbitrary number of signals, there is a research report with two signals for which each shareholder's willingness to pay is not lower. Intuitively, any signal induces one of the three best responses from subscribers: They either vote for the proposal, or they vote against the proposal, or they randomize between voting for and against. In the latter case, the private value of a research report is zero, so the seller is better off designing a research report that avoids such signals. As for the former two types of signals, the seller could equivalently combine all signals that induce subscribers to vote "for" into one, and all signals that induce subscribers to vote "against" into the other. Therefore, it is without loss of generality to limit attention to research reports with two signals. Second, observe that since the seller is the only source of information about the state, one could redefine the problem by referring to signals in the private research reports instead of states. In this redefined problem, an imperfectly informative research report is equivalent to a reduction in each shareholder's payoff from the "correct" decision, which reduces his willingness to pay for the information. Therefore, a fully informative private research report is optimal.

4 Properties of strategic recommendation design

It is interesting to explore the properties of the recommendation designed by the advisor. Consider the case of low μ (the case of high μ is similar). One can measure the degree of bias in recommendations in the following two ways: (1) the factor by which the probability that the signal is $s = 1$ exceeds the probability that the state is $\theta = 1$, which is given by

$$\frac{\Pr(s=1)}{\Pr(\theta=1)} = \frac{\frac{\mu}{\mu_1}}{\mu} = \frac{1 + (1 - 4q^{*2})^{\frac{N-1}{2}}}{(1 - 4q^{*2})^{\frac{N-1}{2}}}, \tag{17}$$

and (2) the difference between $\Pr(s = 1)$ and $\Pr(\theta = 1)$, which is given by

$$\mu \left(\frac{1}{\mu_1} - 1 \right) = \frac{\mu}{(1 - 4q^{*2})^{\frac{N-1}{2}}}. \tag{18}$$

Notice that q^* only depends on the distribution of v_i (the importance of the proposal to shareholders), but is otherwise independent of other parameters of the model, in particular, μ. Thus, the ratio $\frac{\Pr(s=1)}{\Pr(\theta=1)}$ does not depend on prior μ, whereas the difference $\Pr(s = 1) - \Pr(\theta = 1)$ is increasing in μ for $\mu < \mu_l$ and decreasing in μ for $\mu > 1 - \mu_h$.

First, consider the comparative statics with respect to the number of shareholders N. Notice that q^* does not depend on N, and consider the case of $q^* < \frac{1}{2}$. Then, $1 - 4q^{*2} \in (0, 1)$, and therefore $(1 - 4q^{*2})^{\frac{N-1}{2}}$, is decreasing in N, approaching zero for $N \to \infty$. This implies that μ_1 is decreasing in N. Hence, the range of priors over which a (partially) informative public recommendation is optimal shrinks. However, if the prior μ belongs to the region in which a partially informative public recommendation is optimal, the recommendation creates more controversy in the sense that both $\frac{\Pr(s=1)}{\Pr(\theta=1)}$ and $\Pr(s = 1) - \Pr(\theta = 1)$ increase in N for low μ (similarly, both $\frac{\Pr(s=0)}{\Pr(\theta=0)}$ and $\Pr(s = 0) - \Pr(\theta = 0)$ increase in N for high μ). Controversial recommendations become very frequent and thus not very informative.

Second, consider the comparative statics with respect to the fraction of shareholders that subscribe to the research report (q^*). Recall that q^* is the value that maximizes $q\tilde{h}(1 - q)$, where $\tilde{h}(\cdot) = H^{-1}(\cdot)$ is the inverse of the c.d.f. of proposal importance to shareholders. Hence, comparative statics in q^* should be interpreted as comparative statics in properties of the c.d.f. $H(\cdot)$ that determine q^*. Notice that $(1 - 4q^{*2})^{\frac{N-1}{2}}$ is decreasing in q^*, reaching zero for $q^* = \frac{1}{2}$. Therefore, as q^* increases, the range of priors over which the public recommendation provides some information shrinks, and the degree with which it creates controversy increases, in the sense that both $\frac{\Pr(s=1)}{\Pr(\theta=1)}$ and $\Pr(s=1) - \Pr(\theta=1)$ increase in $\hat{q}$ for $\mu < \frac{1}{2}$.

The next proposition summarizes these comparative statics:

Proposition 6. *The extent of recommendations against the prior, measured by either* $\frac{\Pr(s=1)}{\Pr(\theta=1)}$ *or* $\Pr(s=1) - \Pr(\theta=1)$ *when* $\mu < \mu_l$ *(*$\frac{\Pr(s=0)}{\Pr(\theta=0)}$ *or* $\Pr(s=0) - \Pr(\theta=0)$ *when* $\mu > \mu_h$*):*

- *increases in the number of shareholders* N*;*
- *increases with any change in the distribution* $H(\cdot)$ *that increases* $q^* \equiv \arg\max_q q\tilde{h}(1-q)$*.*

5 Extension: Exogenously informed shareholders

The base model makes a stark assumption that the proxy advisor is the only source of information for shareholders. In reality, large institutional investors perform their own research and thus are likely to vote informatively irrespectively of research reports and recommendations of the advisor. Does the presence of these informed shareholders lead a profit-maximizing adviser to design more or less biased recommendations?

To study this question, consider the following simple extension of the base model. Suppose that with probability $\chi \in (0,1)$, a voter is informed about the state without buying the research report, which he learns at the beginning of the game. Suppose that the research report is fully informative about the state, and suppose that the prior μ is sufficiently asymmetric, as in the analysis of Section 3.4. The solution of the model is unchanged up until the seller's problem, which now becomes:

$$\max_{q,S} \sum_{s \in S} (q-\chi) \Pr(Piv|\theta=1, q)\, \tilde{h}(1-q)\, \tau(s) \tag{19}$$

subject to the constraint that the information policy is Bayes plausible:

$$\sum_{s \in S} \mu_s \tau(s) = \mu. \tag{20}$$

Here, $q \geq \chi$ denotes the probability with which a shareholder learns the state, and therefore $q - \chi$ is the probability with which a shareholder subscribes to the seller's report.

Notice that as in the base model, one can consider the problem of the optimal recommendation design for a given q. Thus, the analysis of Section 3.3 and Proposition 3 are unchanged. The only part that is different is the pricing of information by the seller. Specifically, if the seller designs a biased recommendation, it is now optimal to choose q that maximizes

$$\max_{q} (q - \chi) \tilde{h} (1 - q). \tag{21}$$

Denote the solution by $q_e^* (\chi)$. As we show in the proof of the next proposition, for any $\chi > 0$, $q_e^* (\chi)$ satisfies $q_e^* (\chi) > q^*$. In other words, having exogenously informed shareholders leads to a higher fraction of informed shareholders even after accounting for the strategic pricing of information by the seller. Since there are more informed shareholders in equilibrium, the argument in Proposition 6 suggests that profit-maximizing public recommendations become more biased and less informative. The following proposition summarizes this result:

Proposition 7. *Suppose that μ is sufficiently asymmetric and that a partially informative recommendation is optimal. A marginal increase in $\chi \geq 0$ increases $q_e^* (\chi)$ and increases the frequency of recommendations against the prior, measured by either $\frac{\Pr(s=1)}{\Pr(\theta=1)}$ or $\Pr (s = 1) - \Pr (\theta = 1)$ when μ is low ($\frac{\Pr(s=0)}{\Pr(\theta=0)}$ or $\Pr (s = 0) - \Pr (\theta = 0)$ when μ is high).*

6 Implications and discussion

In this section, we relate our results to the empirical evidence on shareholders' voting patterns and their use of proxy advisors, and also discuss several assumptions and policy implications.

6.1 Information content of research reports and recommendations

Our model predicts that the proxy advisor will design research reports that are accurate and unbiased ("fully informative") but that its public recommendations will be partially informative and biased. Consistent with this, many mutual funds that subscribe to proxy advisors and that also invest in stewardship suggest that their prime interest in the feedback from the proxy advisors is the detailed data and reports that they generate, rather than the specific recommendations. For example, at the SEC's Proxy Advisory Services Roundtable in 2013, Michelle Edkins, Global Head of BlackRock's Investment Stewardship team, praised the information content of proxy advisors' research reports stating "we get to read a lot [of proxy statements], and it can be very hard to find the pertinent information ... so having that information synthesized and accessible is hugely important to us being able to take an informed decision," but emphasized that they rely less on specific recommendations and guidelines: "we take our decisions on a case-by-case basis."[17] This view is consistent with a broader view of many other institutional investors, as evidenced by the survey evidence in Bew and Fields (2012) cited in the introduction.

Relatedly, Ertimur, Ferri, and Oesch (2013 and 2018) examine the information content of ISS and Glass Lewis research reports and its relation to shareholders' voting on say-on-pay proposals and director elections, respectively. Their evidence is consistent with shareholders utilizing the contents of the research reports beyond the information contained in the recommendations. In particular, they show that shareholders' tendency to vote against the company's executive compensation policies and its directors is stronger if the research report identifies multiple, rather than a single, reasons for concern, and if the severity of these concerns is higher.

Importantly, even though the advisor's recommendations in our model are biased and less informative than its reports, they nevertheless contain valuable information (whenever the priors are sufficiently asymmetric). Is there evidence that proxy advisors' recommendations are indeed informative about the value of the proposal? Alexander et al. (2010) examine the price impact to ISS recommendations in proxy contests and conclude that the answer is yes. Their analysis suggests that the price impact contains both a *prediction* component (recommendations affect prices by changing the beliefs about who will win the proxy contest) and a *certification* component (recommendations are informative about the value that the dissident or incumbent team would create for the firm); the latter component suggests that recommendations are at least partially informative.

6.2 Shareholders' sensitivity to proxy advisors' recommendations

Our model predicts that the optimally designed advisor's recommendations are biased against the a priori more likely alternative. Empirically, management proposals are typically approved with a high support rate, suggesting that "for management" is often the a priori more likely alternative. For example, the average support rate is about 95% in director elections (Cai, Garner, and Walkling, 2009; Ertimur, Ferri, and Oesch, 2018) and about 90% for

[17]See the transcript of the roundtable at https://www.sec.gov/spotlight/proxy-advisory-services/proxy-advisory-services-transcript.txt.

say-on-pay proposals (Ertimur, Ferri, and Oesch, 2013; Malenko and Shen, 2016). This is expected, given that proposals are endogenously put forward by management and thus are more likely to be designed to appeal to the shareholders.[18]

Given this, our model predicts a particular pattern of shareholders' deviations from proxy advisors' recommendations. In situations where the advisor's recommendation is in favor of the *a priori* expected alternative (in favor of management in the examples above), shareholders should "rubberstamp" this recommendation. In contrast, if the recommendation is against the *a priori* expected alternative, we expect a large disagreement in shareholders' votes, with some shareholders voting in favor and some against.

The evidence on say-on-pay proposals and director elections is consistent with this prediction if we assume that "for management" is indeed the *a priori* more likely alternative. Specifically, Table 1 in Malenko and Shen (2016)'s analysis of say-on-pay votes shows that favorable ISS recommendations are accompanied by 93% average shareholder support and zero failed proposals (out of 1,764 proposals with a positive recommendation in their sample), consistent with the "rubberstamping" prediction. In contrast, negative ISS recommendations are accompanied by 69% average support and an 11% likelihood of the say-on-pay proposal being rejected. Table 1 in Ertimur, Ferri, and Oesch (2018) shows a similar pattern for director elections: all directors that received a favorable recommendation from either ISS or Glass Lewis received majority support (with the vast majority receiving more than 90% votes in favor), but there is much greater dispersion in shareholder votes upon a negative recommendation from either proxy advisor.

6.3 Bias in recommendations and the correct voting benchmark

In our model, the bias in the proxy advisor's recommendations arises because the advisor is maximizing its profits from information sales, rather than the value of the operating companies. One way to explore our conclusions is to compare the recommendations of the proxy advisors with the votes cast by large asset managers, whose interests are potentially more directly aligned with value maximization. Empirical evidence highlights that proxy advisors often make recommendations that are more anti-management than the votes of the major index funds. For example, Brav et al. (2020) show that large index funds, such as BlackRock and Vanguard, seem to be more supportive of management than ISS in proxy

[18] Additionally, many of the votes are about non-contentious matters (such as most director elections).

contests, and the estimates of investor ideology and corporate governance preferences in Bolton et al. (2020) and Bubb and Catan (2019) suggest a similar pattern for other types of proposals and both ISS and Glass Lewis recommendations.

Index funds are frequently criticized because their votes do not align with the proxy advisory recommendations, and their greater support for management is viewed as a sign of passivity or pro-management bias. In contrast, our results suggest a different interpretation of such voting behavior and emphasize that proxy advisors' recommendations may not be the most suitable benchmark (see also discussion in Spatt, 2021). Assuming, given the discussion above, that "for management" is the a priori more likely alternative, shareholders who deviate from negative proxy advisors' recommendations and support the management, could instead be optimally correcting the "controversy" bias in these recommendations.

If proxy advisors' recommendations are not always the correct benchmark, what could be an alternative? Our results suggest that the votes of large asset managers could potentially reflect a more suitable benchmark. To see this, note that our model allows an additional interpretation for v_i, the extent to which shareholders care about the proposal: v_i could stand for the number of firms in a shareholder's portfolio.[19] Under this interpretation, the model predicts that institutional investors that manage larger portfolios are more likely to subscribe to the proxy advisor's reports and make voting decisions based on their own analysis of these reports, rather than purely based on the proxy advisor's recommendations. This prediction is consistent with the idea in Iliev and Lowry (2015), Iliev, Kalodimos, and Lowry (2021), and Gantchev and Giannetti (2020) that larger asset managers are more likely to conduct governance research and be "active voters." Moreover, in our model, the votes of shareholders with large v_i are both informed and unbiased, and thus could be viewed as a more suitable benchmark than the biased recommendations of the proxy advisor.

6.4 One-size-fits-all recommendations

Proxy advisors issue recommendations according to their pre-specified guidelines and often without taking into account the individual circumstances of the company in question (see section 5.1 in Iliev and Lowry (2015) and the field study evidence in Hayne and Vance

[19] Assuming that all firms have the same prior probability that a certain proposal is value-increasing, the shareholders' concern about the value of the proposal, v_i, would be proportional to the number of firms in their portfolio that have this proposal on the agenda.

(2019)). This one-size-fits-all approach has been widely criticized.[20] Our results suggest that such a one-size-fits-all approach can arise naturally, to help the proxy advisor "create controversy" around the issue at vote. Consider, for example, the voting guidelines of proxy advisors on director elections. One of the aspects of these guidelines concerns overboarding. For example, ISS will recommend against "individual directors who sit on more than five public company boards," and Glass Lewis will recommend against "a director who serves as an executive officer of any public company while serving on more than two public company boards and any other director who serves on more than five public company boards."[21] Being overboarded is indeed likely to be on average negatively correlated to the time the director spends with the company and his value contribution (see, e.g., Fich and Shivdasani, 2006). In this sense, recommending against a director who is overboarded provides valuable information, but at the same time is biased against the director, since many busy directors are likely to be valuable given their extensive connections and expertise.[22] Our analysis shows that by designing such one-size-fits-all guidelines, the proxy advisor can generate controversy around director elections, which would induce more shareholders to purchase its research reports and thereby acquire more detailed information about the director's qualifications and experience.[23]

In addition, we would expect subscribers to proxy advisors' reports, especially large asset managers that make substantial investments in due diligence, to deviate from this "one-size-fits-all approach." By reading and internalizing the information in the reports, these shareholders could be accounting for firm-specific factors and often coming to different conclusions from proxy advisors on these issues. This prediction is consistent with the

[20] See, e.g., the discussion at the SEC's 2013 Proxy Advisory Services Roundtable available at https://www.sec.gov/spotlight/proxy-advisory-services/proxy-advisory-services-transcript.txt. See also the comment letter from the directors of 43 open-end and closed-end funds to the SEC: "we are concerned with the practice of at least one proxy advisory firm to implement what is effectively a 'one-size-fits-all' policy that applies one vote recommendation to all similar proxy proposals without analyzing the issue on a company-by-company basis" at http://www.sec.gov/comments/s7-14-10/s71410-196.pdf.

[21] The norms in this area and the nature of the responsibilities of a board member (and compensation) have changed substantially over time as illustrated by https://www.thestreet.com/investing/stocks/the-age-of-vernon-jordan-is-over-serving-on-a-board-is-a-full-time-job-now-13271386. At some point the late Vernon Jordan served on ten boards.

[22] This perspective is illustrated by the comment of a colleague of one of the authors in a broader context: "If you want something done, then you should ask a busy person."

[23] Other aspects of the criticism of a "one-size-fits-all" approach include that the advice would not adjust to reflect other portfolio holdings of the investors (which would be particularly relevant in a merger and acquisition situation) or the tax circumstances of the investor. Since the preferences of shareholders in our model are aligned, we do not capture these other aspects of the criticism.

analysis of one-size-fits-all recommendations in Section 5.1 of Iliev and Lowry (2015), who conclude: "There are important issues on which ISS is predisposed to recommending against management, and active voter mutual funds frequently come to a different conclusion than ISS on these issues."

A specific type of one-size-fits-all recommendations is when proxy advisors always recommend against or in favor of certain proposals. For example, the 2021 ISS guidelines contain a "general recommendation" to "vote against proposals to classify (stagger) the board" and "vote for proposals to repeal classified boards and to elect all directors annually," and Glass Lewis 2021 guidelines state that "Glass Lewis favors the repeal of staggered boards and the annual election of directors." Our analysis predicts the emergence of such recommendations as well. In particular, we show that sometimes the proxy advisor optimally designs completely uninformative recommendations, and always recommending against a certain type of proposals (or always in favor) is one way to implement such uninformative recommendations.

6.5 Discussion

In this subsection, we discuss some institutional features of the proxy advisory process in the context of our model and assumptions.

Multiple proposals and firms. Our baseline model features one firm and one proposal. In practice, proxy advisors sell their research as a bundle: a shareholder subscribing to the proxy advisor receives research reports for all companies in the shareholder's portfolio, and the research report for each company contains information on all proposals on the company's agenda. Such bundling does not change the conclusions of our model, in that the incentives to create controversy arise in this case as well. To see this, suppose that there is still one firm, but it has K proposals on the agenda. The proxy advisor combines its research on all K proposals in one report, so that shareholders choose whether to purchase the report with information on all K proposals, or not purchase any information at all. Importantly, for any of these proposals, the value of the report to the shareholders increases in the probability that the vote on this proposal is close. Hence, for a given fee, the problem of the proxy advisor can be considered as K separate problems, and the incentives to create controversy will arise on any of those K proposals, as long as its prior probability of being value-increasing is sufficiently asymmetric. (Of course, the fee that the advisor will charge for its report will now depend on the combined value of these proposals to the shareholders.) A similar argument applies to a subscription that bundles research across multiple firms in the subscribing shareholders' portfolios.

Communication among shareholders. In our setting, since all shareholders' interests are perfectly aligned, it would be in the ex-post interest of the subscribing shareholders to disclose the information they learn from the advisor's report to other shareholders, and such communication would be credible. For example, even though sharing the report itself is likely not possible given contractual restrictions, the subscribers could nevertheless disclose how they are going to vote. We assume that this does not happen, because in practice, the extent of such disclosure is often limited for several reasons. One reason is that communication with other shareholders could be considered as "forming a group," which may trigger a poison pill or require filing form 13D. For example, the 2011 report by Dechert LLP states that "shareholder concern about unintentionally forming a group has chilled communications among large holders of shares in U.S. public companies."[24] Another reason is that, based on anecdotal evidence, publicly disclosing one's vote against management is viewed by management much more negatively than a negative (but private) vote per se, so institutional investors are usually reluctant to disclose such votes to avoid managerial retaliation.

7 Conclusion

This paper analyzes the information design problem of a proxy advisor that aims to maximize its profits from information sale to voters. The advisor designs two signals – one available to all shareholders for free (a voting recommendation), and one available only to the advisor's subscribers (a research report). The advisor also optimally chooses the fee that it charges for subscribing to its research report.

We show that even though all shareholders are unbiased and aligned at maximizing firm value, the advisor designs recommendations that are biased against the *a priori* more likely alternative. By "creating controversy" in this way, the advisor makes a close vote more likely and, in turn, increases the incentives of shareholders to subscribe to its research report, which it makes fully informative and unbiased. Such design of recommendations is consistent with the "one-size-fits-all approach" for which proxy advisors are frequently

[24] See https://www.jdsupra.com/legalnews/us-court-clarifies-shareholders-actin-14535/.

criticized. Our model predicts that while shareholders will rubberstamp recommendations that support the *a priori* more likely alternative, there will be a lot of disagreement in shareholders' votes upon a recommendation that goes against the more likely alternative. This prediction is consistent with the observed evidence on institutional investors' voting behavior and suggests a reinterpretation of that evidence.

Our paper focuses on a monopolistic proxy advisor, whereas in practice, the proxy advisory industry is a duopoly. Analyzing the joint information design problem of two advisors competing with each other is an interesting direction that we leave for future research.

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Appendix: Proofs

Proof of Proposition 1. Using

$$\begin{aligned}\Pr\left(Piv|\theta=1,q,\mu_s\right) &= C_{N-1}^{\frac{N-1}{2}}\left(\left(q+\left(1-q\right)\pi\right)\left(1-q\right)\left(1-\pi\right)\right)^{\frac{N-1}{2}}, \\ \Pr\left(Piv|\theta=0,q,\mu_s\right) &= C_{N-1}^{\frac{N-1}{2}}\left(\left(1-q\right)\pi\left(1-\pi\left(1-q\right)\right)\right)^{\frac{N-1}{2}},\end{aligned}$$

plugging these expressions into (1), and simplifying, we obtain

$$\Pr\left(\theta=1|q,\mu_s,Piv\right)=\frac{\left(\left(q+\left(1-q\right)\pi\right)\left(1-\pi\right)\right)^{\frac{N-1}{2}}\mu_s}{\left(\left(q+\left(1-q\right)\pi\right)\left(1-\pi\right)\right)^{\frac{N-1}{2}}\mu_s+\left(\pi\left(1-\pi\left(1-q\right)\right)\right)^{\frac{N-1}{2}}\left(1-\mu_s\right)}. \tag{22}$$

First, consider a candidate equilibrium in which each non-subscribing shareholder votes $a_i = 1$. In this case, $\pi = 1$. Therefore, (22) implies that $\Pr(\theta = 1|q, \mu_s, Piv) = 0$ if $\mu_s < 1$, and $\Pr(\theta = 1|q, \mu_s, Piv) = 1$ if $\mu_s = 1$. Hence, this equilibrium exists if and only if $\mu_s = 1$.

Second, consider a candidate equilibrium in which each non-subscribing shareholder votes $a_i = 0$. In this case, $\pi = 0$. Therefore, (22) implies that $\Pr(\theta = 1|q, \mu_s, Piv) = 1$ if $\mu_s > 0$, and $\Pr(\theta = 1|q, \mu_s, Piv) = 0$ if $\mu_s = 0$. Hence, this equilibrium exists if and only if $\mu_s = 0$.

Finally, consider a candidate equilibrium in which each non-subscribing shareholder votes $a_i = 1$ with probability $\pi \in (0, 1)$. For this to be optimal, he must be indifferent between $a_i = 1$ and $a_i = 0$. This is the case if and only if $\Pr(\theta = 1|q, \mu_s, Piv) = \frac{1}{2}$. Equivalently,

$$\left(\left(q+\left(1-q\right)\pi\right)\left(1-\pi\right)\right)^{\frac{N-1}{2}}\mu_s=\left(\pi\left(1-\pi\left(1-q\right)\right)\right)^{\frac{N-1}{2}}\left(1-\mu_s\right). \tag{23}$$

Denoting $z_s \equiv \left(\frac{\mu_s}{1-\mu_s}\right)^{\frac{2}{N-1}}$, we can rewrite this equation as

$$\left(z_s-1\right)\left(1-q\right)\pi^2+\left(1+\left(2q-1\right)z_s\right)\pi-z_sq=0. \tag{24}$$

It has a unique positive root given by (2). Since the left-hand side of (24) is negative at $\pi = 0$ and $q > 0$ at $\pi = 1$, this root is between 0 and 1.

Proof of Proposition 2. Proven in the main text.

Proof of Proposition 3. We can rewrite the indifference condition (23) as

$$\Pr\left(Piv|\theta=1,q,\mu\right)\mu=\Pr\left(Piv|\theta=0,q,\mu\right)\left(1-\mu\right).$$

Therefore,

$$\begin{aligned}\Pr\left(Piv|q,\mu\right) &= \Pr\left(Piv|\theta=1,q,\mu\right)\mu+\Pr\left(Piv|\theta=0,q,\mu\right)\left(1-\mu\right) \\ &= 2\mu C_{N-1}^{\frac{N-1}{2}}\left(\left(q+\left(1-q\right)\pi\left(q,\mu\right)\right)\left(1-q\right)\left(1-\pi\left(q,\mu\right)\right)\right)^{\frac{N-1}{2}},\end{aligned} \tag{25}$$

where $\pi(q, \mu)$ is the equilibrium probability that a non-subscribing shareholder votes "for," given by (2). Plugging (2) from Proposition 1 into (25) and simplifying the expression, we get

$$\Pr\left(Piv|q,\mu\right) = 2C_{N-1}^{\frac{N-1}{2}} q^{\frac{N-1}{2}} \left(\frac{\sqrt{\left(z\left(\mu\right)-1\right)^2 + 4q^2 z\left(\mu\right)} - q\left(1+z\left(\mu\right)\right)}{\left(z\left(\mu\right)-1\right)^2}\right)^{\frac{N-1}{2}} \mu,$$

where $z\left(\mu\right) \equiv \left(\frac{\mu}{1-\mu}\right)^{\frac{2}{N-1}}$. Define $\Omega\left(\mu|q\right)$:

$$\Omega\left(\mu|q\right) \equiv \left(\frac{\varphi\left(z\left(\mu\right)\right)}{\left(z\left(\mu\right)-1\right)^2}\right)^{\frac{N-1}{2}} \mu, \tag{26}$$

where

$$\varphi\left(z\right) \equiv \sqrt{\left(z-1\right)^2 + 4q^2 z} - q\left(1+z\right). \tag{27}$$

Then, $\Pr\left(Piv|q,\mu\right)$ is concave (convex) in μ at some $\left(\mu,q\right)$ if and only if $\Omega\left(\mu|q\right)$ is concave (convex) in μ at this $\left(\mu,q\right)$. Taking the first and second derivative of $z\left(\mu\right)$ and the first four derivatives of $\varphi\left(z\right)$:

$$z'\left(\mu\right) = \frac{2}{N-1}\frac{z\left(\mu\right)}{\mu\left(1-\mu\right)}, \tag{28}$$

$$z''\left(\mu\right) = \frac{2}{N-1}\frac{\frac{2}{N-1}-\left(1-2\mu\right)}{\mu^2\left(1-\mu\right)^2} z\left(\mu\right), \tag{29}$$

$$\varphi\prime\left(z\right) = \frac{z-1+2q^2}{\sqrt{\left(z-1\right)^2+4q^2 z}} - q, \tag{30}$$

$$\varphi''\left(z\right) = \frac{4q^2\left(1-q^2\right)}{\left(\left(z-1\right)^2+4q^2 z\right)^{\frac{3}{2}}} \tag{31}$$

$$\varphi'''\left(z\right) = \frac{-12q^2\left(1-q^2\right)\left(z-1+2q^2\right)}{\left(\left(z-1\right)^2+4q^2 z\right)^{\frac{5}{2}}} \tag{32}$$

$$\varphi''''\left(z\right) = \frac{60q^2\left(1-q^2\right)\left(z-1+2q^2\right)^2}{\left(\left(z-1\right)^2+4q^2 z\right)^{\frac{7}{2}}} - \frac{12q^2\left(1-q^2\right)}{\left(\left(z-1\right)^2+4q^2 z\right)^{\frac{5}{2}}} \tag{33}$$

Differentiating (26) twice,

$$\begin{aligned}\Omega''\left(\mu|q\right) = {} & \Omega'\left(\mu|q\right)\left(\frac{N-1}{2}\left(\frac{\varphi'\left(z\right)}{\varphi\left(z\right)} - \frac{2}{z-1}\right) z'\left(\mu\right) + \frac{1}{\mu}\right) \\ & + \Omega\left(\mu|q\right)\left(\frac{N-1}{2}\left(\left[\frac{\varphi'\left(z\right)}{\varphi\left(z\right)}\right]' + \frac{2}{\left(z-1\right)^2}\right)\left(z'\left(\mu\right)\right)^2 + \frac{N-1}{2}\left(\frac{\varphi'\left(z\right)}{\varphi\left(z\right)} - \frac{2}{z-1}\right) z''\left(\mu\right) - \frac{1}{\mu^2}\right).\end{aligned} \tag{34}$$

Using (28)-(29) and simplifying,

$$\frac{\Omega''(\mu|q)\,\mu}{\Omega(\mu|q)}\mu^{\frac{N-3}{N-1}} = \left(\begin{array}{c}\left(\frac{\varphi'(z(\mu))}{\varphi(z(\mu))}-\frac{2}{z(\mu)-1}\right)^2+\\ +\frac{2}{N-1}\left(\frac{\varphi''(z(\mu))\varphi(z(\mu))-(\varphi'(z(\mu)))^2}{\varphi(z(\mu))^2}+\frac{2}{(z(\mu)-1)^2}\right)\end{array}\right)\mu^{\frac{2}{N-1}}(1-\mu)^{-\frac{2(N+1)}{N-1}}$$
$$+2\left(\frac{\varphi'(z(\mu))}{\varphi(z(\mu))}-\frac{2}{z(\mu)-1}\right)(1-\mu)^{-\frac{N+1}{N-1}}$$
$$+\left(\frac{\varphi'(z(\mu))}{\varphi(z(\mu))}-\frac{2}{z(\mu)-1}\right)\left(\frac{2}{N-1}-(1-2\mu)\right)(1-\mu)^{-\frac{2}{N}}.$$

Consider the limit case of $\mu \to 0$. By symmetry of $\Omega(\mu|q)$ around $\mu = \frac{1}{2}$, the case of $\mu \to 1$ is identical. When $\mu \to 0$, $\lim_{\mu\to 0} z(\mu) = 0$, $\lim_{\mu\to 0}\varphi(z) = 1-q$, and $\lim_{\mu\to 0}\varphi\prime(z) = (q-1)(2q+1)$. Therefore,

$$\lim_{\mu\to 0}\frac{\Omega''(\mu|q)\,\mu^{\frac{2N-4}{N-1}}}{\Omega(\mu|q)} = \frac{N+1}{N-1}(1-2q).$$

Hence, $\Omega''(\mu|q) > 0$ for μ close to 0 if and only if $q < \frac{1}{2}$.

Next, consider the limit case of $\mu \to \frac{1}{2}$. Using $\Omega'\left(\frac{1}{2}|q\right) = 0$ and the expressions (28)-(29), (34) at $\mu = \frac{1}{2}$ yields

$$\frac{\Omega''\left(\frac{1}{2}|q\right)}{\Omega\left(\frac{1}{2}|q\right)} = \frac{32}{N-1}\lim_{z\to 1}\left(\frac{\varphi''(z)\varphi(z)-(\varphi'(z))^2}{\varphi(z)^2}+\frac{2}{(z-1)^2}\right)+\frac{32}{N-1}\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)-4.$$

Consider the second limit. Notice that $\varphi(1) = \varphi'(1) = 0$ and $\varphi''(1) \neq 0$. Applying l'Hopital's rule three times,

$$\begin{aligned}\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right) &= \lim_{z\to 1}\left(\frac{\varphi'(z)(z-1)-2\varphi(z)}{\varphi(z)(z-1)}\right)\\ &= \lim_{z\to 1}\left(\frac{\varphi''''(z)(z-1)+\varphi'''(z)}{\varphi'''(z)(z-1)+3\varphi''(z)}\right)\\ &= \frac{\varphi'''(1)}{3\varphi''(1)} = -\frac{1}{2},\end{aligned}$$

where the last transition is from evaluating (31) and (32) at $z = 1$. Consider the first limit. Using $\lim_{z\to 1}\left(\frac{\varphi'(z)}{\varphi(z)}-\frac{2}{z-1}\right)^2 = \frac{1}{4}$ and applying l'Hopital's rule four times,

$$\begin{aligned}\lim_{z\to 1}\left(\frac{\varphi''(z)\varphi(z)-(\varphi'(z))^2}{\varphi(z)^2}+\frac{2}{(z-1)^2}\right) &= \lim_{z\to 1}\left(\frac{\varphi''(z)(z-1)^2+6\varphi(z)-4\varphi'(z)(z-1)}{\varphi(z)(z-1)^2}\right)-\frac{1}{4}\\ &= \lim_{z\to 1}\left(\frac{\varphi''''''(z)(z-1)^2+4(z-1)\varphi'''''(z)+2\varphi''''(z)}{\varphi''''(z)(z-1)^2+8(z-1)\varphi'''(z)+12\varphi''(z)}\right)-\frac{1}{4}\\ &= \frac{\varphi''''(1)}{6\varphi''(1)}-\frac{1}{4} = \frac{\frac{3(1-q^2)(5q^2-1)}{8q^3}}{6\frac{1-q^2}{2q}}-\frac{1}{4} = \frac{3q^2-1}{8q^2},\end{aligned}$$

where the transition on the last line is from evaluating (31) and (33) at $z = 1$. Therefore,

$$\begin{aligned}\frac{\Omega''\left(\frac{1}{2}|q\right)}{\Omega\left(\frac{1}{2}|q\right)} &= \frac{32}{N-1}\frac{3q^2-1}{8q^2} - \frac{16}{N-1} - 4 \\ &= -\frac{4}{N-1}\frac{1+q^2}{q^2} - 4 < 0.\end{aligned}$$

Hence, $\Pr\left(Piv|q,\mu\right)$ is strictly concave at $\mu = \frac{1}{2}$, and, by continuity of the second derivative, also in the neighborhood of $\mu = \frac{1}{2}$.

Proof of Proposition 4. The main argument is described in the text. Here we fill in the remaining details. Notice that (10) depends on m only via $t = q + (1-q)\,\pi(q,m)$, and the dependence is of the form $(t(1-t))^{\frac{N-1}{2}}$. Therefore, if feasible, the optimal m is such that $t = \frac{1}{2}$. Plugging in the expression for $\pi(q,m)$ from (2):

$$\begin{aligned}(1-q)\left(1-\pi(q,m)\right) &= \frac{1}{2} + \frac{2q - \sqrt{(z-1)^2 + 4q^2 z}}{2(z-1)} = \frac{1}{2} \\ &\Rightarrow\ z^2 - 2\left(1-2q^2\right)z + 1 - 4q^2 = 0,\end{aligned} \tag{35}$$

where $z \equiv \left(\frac{m}{1-m}\right)^{\frac{2}{N-1}}$. Eq. (35) is a quadratic equation, whose only positive root is $z = 1 - 4q^2$. Equating it to $\left(\frac{m}{1-m}\right)^{\frac{2}{N-1}}$ implies that the optimal posterior is (13).

In the example with the power distribution, $H(x) = x^\alpha$, so

$$q = 1 - \left(\frac{f}{V(q,\mathcal{S})}\right)^\alpha \Rightarrow f = (1-q)^{\frac{1}{\alpha}}\, V(q,\mathcal{S}).$$

Thus, the maximization problem for q^* is $\max_q q(1-q)^{\frac{1}{\alpha}}$, yielding

$$1 - q - \frac{1}{\alpha}q^* = 0\ \Rightarrow q^* = \frac{\alpha}{\alpha+1}.$$

Proof of Proposition 5. By contradiction, suppose there is an imperfectly informative research report that is optimal. Then, there must be μ_s and q at which a shareholder's willingness to pay for the imperfectly informative research report is higher than for a fully revealing research report.

Consider an arbitrary research report $\mathcal{R}$. Divide the set of signals R into subsets

$$\begin{aligned}R_0 &\equiv \left\{r \in R : \Pr\left(a(r,\mu_s) = 0\right) = 1\right\}, \\ R_1 &\equiv \left\{r \in R : \Pr\left(a(r,\mu_s) = 1\right) = 1\right\}, \\ R_m &\equiv \left\{r \in R : \Pr\left(a(r,\mu_s) = 1\right) \in (0,1)\right\}.\end{aligned}$$

Intuitively, R_0 (R_1) is the set of signals in the research report that induces all subscribers to vote for "against" ("for"), and R_m is the set of signals in the research report that induces subscribers to mix between voting "for" and "against." Let $W(\mathcal{R},\mu_s,q)$ denote the value of research report $\mathcal{R}$

conditional on μ_s and q. The value of the research report conditional on $r \in R_m$ is zero, since a subscriber is indifferent between voting for and against in this case. Therefore,

$$\begin{aligned} W(\mathcal{R}, \mu_s, q) &= \sum_{r \in R_0} \Pr(r|\mu_s) \Pr(Piv|q, r, \mu_s) \left(\Pr(\theta = 0|r, \mu_s) - \frac{1}{2} \right) \\ &+ \sum_{r \in R_1} \Pr(r|\mu_s) \Pr(Piv|q, r, \mu_s) \left(\Pr(\theta = 1|r, \mu_s) - \frac{1}{2} \right). \end{aligned}$$

Since for any $r \in R_0$ the voting strategy of subscribers is the same, $\Pr(Piv|q, r_1, \mu_s) = \Pr(Piv|q, r_2, \mu_s)$ for any $r_1, r_2 \in R_0$. Similarly, $\Pr(Piv|q, r_1, \mu_s) = \Pr(Piv|q, r_2, \mu_s)$ for any $r_1, r_2 \in R_1$. Therefore,

$$\begin{aligned} W(\mathcal{R}, \mu_s, q) &= \Pr(Piv|q, r \in R_0, \mu_s) \Pr(r \in R_0|\mu_s) \left(\Pr(\theta = 0|r \in R_0, \mu_s) - \frac{1}{2} \right) \\ &+ \Pr(Piv|q, r \in R_1, \mu_s) \Pr(r \in R_1|\mu_s) \left(\Pr(\theta = 1|r \in R_1, \mu_s) - \frac{1}{2} \right). \end{aligned}$$

First, notice that it is without loss of generality to combine all $r \in R_0$ into one signal, denoted $r = 0$, and all $r \in R_1$ into the other signal, denoted $r = 1$. Second, notice that all else equal $W(\mathcal{R}, \mu_s, q)$ is higher if $\Pr(r = 0|\mu_s) + \Pr(r = 1|\mu_s) = 1$, i.e, if set R_m is empty. Thus, we can focus on binary signals.

Denote $p_1 \equiv \Pr(\theta = 1|r = 1, \mu_s)$ and $p_0 = \Pr(\theta = 0|r = 0, \mu_s)$. Also, denote $P(x) = C_{N-1}^{\frac{N-1}{2}} (x(1-x))^{\frac{N-1}{2}}$. We have:

$$W(\mathcal{R}, \mu_s, q) = P((1-q)\pi) \Pr(r = 0|\mu_s) \left(p_0 - \frac{1}{2} \right) + P(q + (1-q)\pi) \Pr(r = 1|\mu_s) \left(p_1 - \frac{1}{2} \right)$$

Bayes' rule implies:

$$\begin{aligned} p_1 \Pr(r = 1|\mu_s) + (1 - p_0) \Pr(r = 0|\mu_s) &= \mu_s \\ \Pr(r = 1|\mu_s) + \Pr(r = 0|\mu_s) &= 1, \end{aligned}$$

or

$$\begin{aligned} \Pr(r = 1|\mu_s) &= \frac{\mu_s - (1 - p_0)}{p_1 + p_0 - 1} \\ \Pr(r = 0|\mu_s) &= \frac{p_1 - \mu_s}{p_1 + p_0 - 1} \end{aligned}$$

Hence,

$$W(\mathcal{R}, \mu_s, q) = P((1-q)\pi) \frac{p_1 - \mu_s}{p_1 + p_0 - 1} \left(p_0 - \frac{1}{2} \right) + P(q + (1-q)\pi) \frac{\mu_s - (1 - p_0)}{p_1 + p_0 - 1} \left(p_1 - \frac{1}{2} \right). \tag{36}$$

Next, we have the indifference condition of non-subscribers:

$$\Pr(r = 1|q, \mu_s, Piv) \Pr(\theta = 1|r = 1, \mu_s) = \Pr(r = 0|q, \mu_s, Piv) \Pr(\theta = 0|r = 0, \mu_s),$$

where

$$\Pr\left(r=1|q,\mu_s,Piv\right)=\frac{\frac{\Pr(Piv_i|r=1,q,\mu_s)}{\Pr(Piv_i|r=0,q,\mu_s)}\frac{\Pr(r=1|\mu_s)}{\Pr(r=0|\mu_s)}}{\frac{\Pr(Piv_i|r=1,q,\mu_s)}{\Pr(Piv_i|r=0,q,\mu_s)}\frac{\Pr(r=1|\mu_s)}{\Pr(r=0|\mu_s)}+1}.$$

Rewriting:

$$\begin{aligned}\frac{\Pr\left(r=1|q,\mu_s,Piv\right)}{\Pr\left(r=0|q,\mu_s,Piv\right)} &= \frac{\Pr\left(Piv_i|r=1,q,\mu_s\right)}{\Pr\left(Piv_i|r=0,q,\mu_s\right)}\frac{\Pr\left(r=1|\mu_s\right)}{\Pr\left(r=0|\mu_s\right)}=\frac{\Pr\left(\theta=0|r=0,\mu_s\right)}{\Pr\left(\theta=1|r=1,\mu_s\right)}\\ \frac{\Pr\left(Piv_i|r=1,q,\mu_s\right)}{\Pr\left(Piv_i|r=0,q,\mu_s\right)}\frac{\Pr\left(r=1\right)}{\Pr\left(r=0\right)} &= \frac{p_0}{p_1}.\end{aligned}$$

Notice that for a given ratio $\frac{p_0}{p_1}$, it is optimal to increase p_0 and p_1 as much as possible. Thus, either $p_1=1$ and $p_0\le 1$, or $p_0=1$ and $p_1\le 1$.

Consider the case of $p_0=1$ and $p_1\le 1$. The case of $p_1=1$ and $p_0\le 1$ is analogous by symmetry. Then, (36) simplifies to

$$W\left(\mathcal{R},\mu_s,q\right)=P\left(\left(1-q\right)\pi\right)\left(1-\frac{\mu_s}{p_1}\right)\frac{1}{2}+P\left(q+\left(1-q\right)\pi\right)\mu_s\left(1-\frac{1}{2p_1}\right).$$

Consider the indifference condition of non-subscribers:

$$P\left(\left(1-q\right)\pi\right)\left(1-\frac{\mu_s}{p_1}\right)=P\left(q+\left(1-q\right)\pi\right)\mu_s.$$

Then,

$$W\left(\mathcal{R},\mu_s,q\right)=P\left(q+\left(1-q\right)\pi\right)\mu_s\left(\frac{3}{2}-\frac{1}{2p_1}\right)$$

Differentiate with respect to p_1:

$$P'\left(q+\left(1-q\right)\pi\right)\frac{d\pi}{dp_1}\mu_s\left(\frac{3}{2}-\frac{1}{2p_1}\right)+P\left(q+\left(1-q\right)\pi\right)\mu_s\frac{1}{2p_1^2}$$

Note that

$$\pi=\frac{z_s\left(1-2q\right)-1+\sqrt{\left(z_s-1\right)^2+4q^2z_s}}{2\left(z_s-1\right)\left(1-q\right)},$$

where $z_s\equiv\left(\frac{p_1\mu_s}{p_1-\mu_s}\right)^{\frac{2}{N-1}}$. Hence,

$$q+\left(1-q\right)\pi=\frac{1}{2}+\frac{\sqrt{\left(z_s-1\right)^2+4q^2z_s}-2q}{2\left(z_s-1\right)}>\frac{1}{2}.$$

Hence, since $P\left(x\right)$ is inverted U-shaped with the maximum at $x=\frac{1}{2}$, $P'\left(q+\left(1-q\right)\pi\right)<0$. Furthermore,

$$d\left[\frac{p_1\mu_s}{p_1-\mu_s}\right]/dp_1=-\frac{\mu_s^2}{\left(p_1-\mu_s\right)^2}<0.$$

Therefore $\frac{d\pi}{dp_1} = \frac{d\pi}{dz_s}\frac{dz_s}{d\mu_s} < 0$. Therefore,

$$P'\left(q + (1-q)\pi\right)\frac{d\pi}{dp_1}\mu_s\left(\frac{3}{2} - \frac{1}{2p_1}\right) + P\left(q + (1-q)\pi\right)\mu_s\frac{1}{2p_1^2} > 0.$$

Hence, $p_1 = 1$ is optimal.

Proof of Proposition 6. Proven in the main text.

Proof of Proposition 7. We apply the Topkis's theorem (Topkis, 1978) to (21). The cross-partial derivative of the objective function $(q - \chi)\tilde{h}(1-q)$ in q and χ is $\tilde{h}'(1-q) > 0$. Therefore, function $(q-\chi)\tilde{h}(1-q)$ is submodular. Therefore, by Topkis's theorem, $q_e^*(\chi)$ is increasing in χ. Hence, both the ratio (17) and the difference (18) of the frequency of recommendations against the prior (relative to the probability that the state is different from the prior) increase.