1. The percentage change in the bond’s price is:

\[
\frac{-\text{Duration}}{1 + y} \times \Delta y = -\frac{7.194}{1.10} \times 0.005 = -0.0327 = -3.27\% \text{ or a } 3.27\% \text{ decline.}
\]

2. a. YTM = 6%

<table>
<thead>
<tr>
<th>Time until Payment (years)</th>
<th>Cash Flow</th>
<th>PV of CF (Discount rate = 6%)</th>
<th>Weight</th>
<th>Column (1) × Column (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$60.00</td>
<td>$56.60</td>
<td>0.0566</td>
<td>0.0566</td>
</tr>
<tr>
<td>2</td>
<td>$60.00</td>
<td>$53.40</td>
<td>0.0534</td>
<td>0.1068</td>
</tr>
<tr>
<td>3</td>
<td>$1,060.00</td>
<td>$890.00</td>
<td>0.8900</td>
<td>2.6700</td>
</tr>
<tr>
<td>Column Sums</td>
<td>$1,000.00</td>
<td></td>
<td>1.0000</td>
<td>2.8334</td>
</tr>
</tbody>
</table>

Duration = 2.833 years

b. YTM = 10%

<table>
<thead>
<tr>
<th>Time until Payment (years)</th>
<th>Cash Flow</th>
<th>PV of CF (Discount rate = 10%)</th>
<th>Weight</th>
<th>Column (1) × Column (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$60.00</td>
<td>$54.55</td>
<td>0.0606</td>
<td>0.0606</td>
</tr>
<tr>
<td>2</td>
<td>$60.00</td>
<td>$49.40</td>
<td>0.0551</td>
<td>0.1102</td>
</tr>
<tr>
<td>3</td>
<td>$1,060.00</td>
<td>$796.39</td>
<td>0.8844</td>
<td>2.6532</td>
</tr>
<tr>
<td>Column Sums</td>
<td>$900.53</td>
<td></td>
<td>1.0000</td>
<td>2.8240</td>
</tr>
</tbody>
</table>

Duration = 2.824 years, which is less than the duration at the YTM of 6%.

3. For a semiannual 6% coupon bond selling at par, we use the following parameters: coupon = 3% per half-year period, y = 3%, T = 6 semiannual periods. Using Rule 8, we find:

\[
D = \frac{(1.03/0.03)}{0.03} \times [1 - (1/1.03)^6] = 5.58 \text{ half-year periods} = 2.79 \text{ years}
\]

If the bond’s yield is 10%, use Rule 7, setting the semiannual yield to 5%, and semiannual coupon to 3%:

\[
D = \frac{1.05}{0.05} \times \frac{[6 \times (0.03 - 0.05)]}{0.03 \times [(1.05)^6 - 1] + 0.05} = 21 - 15.4478 = 5.5522 \text{ half-year periods} = 2.7761 \text{ years}
\]
4. a. Bond B has a higher yield to maturity than bond A since its coupon payments and maturity are equal to those of A, while its price is lower. (Perhaps the yield is higher because of differences in credit risk.) Therefore, the duration of Bond B must be shorter.

b. Bond A has a lower yield and a lower coupon, both of which cause Bond A to have a longer duration than Bond B. Moreover, A cannot be called, so that its maturity is at least as long as that of B, which generally increases duration.

5. a. 

<table>
<thead>
<tr>
<th>Time until Payment (years)</th>
<th>Cash Flow (1)</th>
<th>PV of CF (Discount rate = 10%) (2)</th>
<th>Weight (4)</th>
<th>Column (1) $ \times $ (Column (4)) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10 million</td>
<td>$9.09 million</td>
<td>0.7857</td>
<td>0.7857</td>
</tr>
<tr>
<td>5</td>
<td>$4 million</td>
<td>$2.48 million</td>
<td>0.2143</td>
<td>1.0715</td>
</tr>
<tr>
<td>Column Sums</td>
<td>$11.57 million</td>
<td></td>
<td>1.0000</td>
<td>1.8572</td>
</tr>
</tbody>
</table>

D = 1.8572 years = required maturity of zero coupon bond.

b. The market value of the zero must be $11.57 million, the same as the market value of the obligations. Therefore, the face value must be:

\[
$11.57 \text{ million} \times (1.10)^{1.8572} = $13.81 \text{ million}
\]

6. a. The call feature provides a valuable option to the issuer, since it can buy back the bond at a specified call price even if the present value of the scheduled remaining payments is greater than the call price. The investor will demand, and the issuer will be willing to pay, a higher yield on the issue as compensation for this feature.

b. The call feature reduces both the duration (interest rate sensitivity) and the convexity of the bond. If interest rates fall, the increase in the price of the callable bond will not be as large as it would be if the bond were noncallable. Moreover, the usual curvature that characterizes price changes for a straight bond is reduced by a call feature. The price-yield curve (see Figure 16.6) flattens out as the interest rate falls and the option to call the bond becomes more attractive. In fact, at very low interest rates, the bond exhibits negative convexity.
7. In each case, choose the longer-duration bond in order to benefit from a rate decrease.
   a. The Aaa-rated bond will have the lower yield to maturity and therefore the longer duration.
   b. The lower-coupon bond will have the longer duration and greater de facto call protection.
   c. Choose the lower coupon bond for its longer duration.

8. a. (iv) \[10 \times 0.01 \times 800 = 80.00\]
   b. (ii) \[\frac{1}{2} \times 120 \times (0.015)^2 = 0.0135 = 1.35\%\]
   c. (i)
   d. (i) \[9/1.10 = 8.18\]
   e. (iii)
   f. (i)
   g. (i)
   h. (iii)

9. The table below shows the holding period returns for each of the three bonds

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM at beginning of year</td>
<td>7.00%</td>
<td>8.00%</td>
<td>9.00%</td>
</tr>
<tr>
<td>Beginning of year prices</td>
<td>$1,009.35</td>
<td>$1,000.00</td>
<td>$974.69</td>
</tr>
<tr>
<td>Prices at year end (at 9% YTM)</td>
<td>$1,000.00</td>
<td>$990.83</td>
<td>$982.41</td>
</tr>
<tr>
<td>Capital gain</td>
<td>$70.65</td>
<td>$70.83</td>
<td>$87.72</td>
</tr>
<tr>
<td>Coupon</td>
<td>$80.00</td>
<td>$80.00</td>
<td>$80.00</td>
</tr>
<tr>
<td>1-year total $ return</td>
<td>$70.65</td>
<td>$70.83</td>
<td>$87.72</td>
</tr>
<tr>
<td>1-year total rate of return</td>
<td>7.00%</td>
<td>7.08%</td>
<td>9.00%</td>
</tr>
</tbody>
</table>

You should buy the 3-year bond because it provides a 9% holding-period return over the next year, which is greater than the return on either of the other bonds.

10. a. Modified duration = \[
        \frac{\text{Macaulay duration}}{1 + \text{YTM}} = \frac{10}{1.08} = 9.26 \text{ years}
    \]
   b. For option-free coupon bonds, modified duration is a better measure of the bond’s sensitivity to changes in interest rates. Maturity considers only the final cash flow, while modified duration includes other factors, such as the size and timing of coupon payments, and the level of interest rates (yield to maturity). Modified duration, unlike maturity, indicates the approximate percentage change in the bond price for a given change in yield to maturity.
c. i. Modified duration increases as the coupon decreases.
   ii. Modified duration decreases as maturity decreases.

d. Convexity measures the curvature of the bond’s price-yield curve. Such
   curvature means that the duration rule for bond price change (which is based
   only on the slope of the curve at the original yield) is only an approximation.
   Adding a term to account for the convexity of the bond increases the accuracy
   of the approximation. That convexity adjustment is the last term in the
   following equation:
   \[
   \Delta P = (-D \times \Delta y) + \left[ \frac{1}{2} \times \text{Convexity} \times (\Delta y)^2 \right]
   \]

11. a. PV of the obligation = [$10,000 \times \text{Annuity factor (8\%, 2)}] = $17,832.65
   Duration = 1.4808 years, which can be verified using Rule 6 or a table such as
   Table 15.3.

   b. A zero-coupon bond maturing in 1.4808 years would immunize the
   obligation. Since the present value of the zero-coupon bond must be
   $17,832.65, the face value (i.e., the future redemption value) must be:
   $17,832.65 \times 1.08^{1.4808} = $19,985.26

   c. If the interest rate increases to 9\%, the zero-coupon bond would decrease in
   value to:
   \[
   \frac{$19,985.26}{1.09^{1.4808}} = $17,950.92
   \]
   The present value of the tuition obligation would decrease to $17,591.11. The
   net position decreases in value by $0.19.

   If the interest rate decreases to 7\%, the zero-coupon bond would increase in
   value to:
   \[
   \frac{$19,985.26}{1.07^{1.4808}} = $18,079.99
   \]
   The present value of the tuition obligation would increase to $18,080.18. The
   net position decreases in value by $0.19.

   The reason the net position changes at all is that, as the interest rate changes,
   so does the duration of the stream of tuition payments.
12. a. In an interest rate swap, one firm exchanges or “swaps” a fixed payment for a payment that is tied to the level of interest rates. One party in the swap agreement pays a fixed interest rate on the notional principal of the swap. The other party pays the floating interest rate (typically LIBOR) on the same notional principal. For example, in a swap with a fixed rate of 8% and notional principal of $100 million, the net cash payment for the firm that pays the fixed rate and receives the floating rate would be: \( (\text{LIBOR} – 0.08) \times \$100 \text{ million} \). Therefore, if LIBOR exceeds 8%, the firm receives money; if LIBOR is less than 8%, the firm pays money.

b. There are several applications of interest rate swaps. For example, a portfolio manager who is holding a portfolio of long-term bonds, but is worried that interest rates might increase, causing a capital loss on the portfolio, can enter a swap to pay a fixed rate and receive a floating rate. The portfolio manager thereby converts the holdings into a synthetic floating rate portfolio. Or, a pension fund manager might identify money market securities that are paying excellent yields compared to other comparable-risk short-term securities. However, the manager might believe that such short-term assets are inappropriate for the portfolio. The fund can hold these securities and enter a swap in which the fund receives a fixed rate and pays a floating rate. Thus, the fund captures the benefit of the advantageous relative yields on these securities, but still establishes a portfolio with interest-rate risk characteristics more like those of long-term bonds.

13. The answer depends on the nature of the long-term assets that the corporation holds. If those assets produce a return that varies with short-term interest rates, then an interest-rate swap would not be appropriate. If, however, the long-term assets are fixed-rate financial assets such as fixed-rate mortgages, then a swap might be risk reducing. In such a case the corporation would swap its floating-rate bond liability for a fixed-rate long-term liability.

14. a. (i) Current yield = \( \frac{\text{Coupon}}{\text{Price}} = \frac{70}{960} = 0.0729 = 7.29\% \)

(ii) YTM = 3.993% semiannually or 7.986% annual bond equivalent yield. On a financial calculator, enter: \( n = 10; \ PV = \ –960; \ FV = 1000; \ PMT = 35 \) Compute the interest rate.
(iii) Realized compound yield is 4.166% (semiannually), or 8.332% annual bond equivalent yield. To obtain this value, first find the future value (FV) of reinvested coupons and principal. There will be six payments of $35 each, reinvested semiannually at of 3% per period. On a financial calculator, enter: PV = 0; PMT = $35; n = 6; i = 3%. Compute: FV = $226.39

Three years from now, the bond will be selling at the par value of $1,000 because the yield to maturity is forecast to equal the coupon rate. Therefore, total proceeds in three years will be $1,226.39.

Then find the rate \( \left( y_{realized} \right) \) that makes the FV of the purchase price equal to $1,226.39:

\[
\$960 \times (1 + y_{realized})^6 = \$1,226.39 \Rightarrow y_{realized} = 4.166\% \text{ (semiannual)}
\]

b. Shortcomings of each measure:
   (i) Current yield does not account for capital gains or losses on bonds bought at prices other than par value. It also does not account for reinvestment income on coupon payments.
   (ii) Yield to maturity assumes the bond is held until maturity and that all coupon income can be reinvested at a rate equal to the yield to maturity.
   (iii) Realized compound yield is affected by the forecast of reinvestment rates, holding period, and yield of the bond at the end of the investor's holding period.

Note: This criticism of horizon return is a bit unfair: while YTM can be calculated without explicit assumptions regarding future YTM and reinvestment rates, you implicitly assume that these values equal the current YTM if you use YTM as a measure of expected return.

15. The firm should enter a swap in which the firm pays a 7% fixed rate and receives LIBOR on $10 million of notional principal. Its total payments will be as follows:

   Interest payments on bond \( (\text{LIBOR} + 0.01) \times \$10 \text{ million par value} \)
   Net cash flow from swap \( (0.07 - \text{LIBOR}) \times \$10 \text{ million notional principal} \)
   Total \( 0.08 \times \$10 \text{ million} \)

   The interest rate on the synthetic fixed-rate loan is 8%.

16. a. PV of obligation = \$2 million/0.16 = \$12.5 million
   Duration of obligation = 1.16/0.16 = 7.25 years
   Call \( w \) the weight on the 5-year maturity bond (which has duration of 4 years).
   Then:
   \[
   (w \times 4) + [(1 - w) \times 11] = 7.25 \Rightarrow w = 0.5357
   \]
   Therefore: \( 0.5357 \times \$12.5 = \$6.7 \text{ million in the 5-year bond, and;} \)
   \( 0.4643 \times \$12.5 = \$5.8 \text{ million in the 20-year bond.} \)
b. The price of the 20-year bond is:

\[ [$60 \times \text{Annuity factor (16\%, 20))] + [$1,000 \times \text{PV factor (16\%, 20)}] = $407.12 \]

Therefore, the bond sells for 0.4071 times its par value, and:

\[
\text{Market value} = \text{Par value} \times 0.4071
\]

\[
$5.8 \text{ million} = \text{Par value} \times 0.4071 \implies \text{Par value} = $14.25 \text{ million}
\]

Another way to see this is to note that each bond with par value $1,000 sells for $407.12. If total market value is $5.8 million, then you need to buy approximately 14,250 bonds, resulting in total par value of $14.25 million.

17. a. The duration of the perpetuity is: 1.05/0.05 = 21 years

Call w the weight of the zero-coupon bond. Then:

\[
(w \times 5) + [(1 - w) \times 21] = 10 \implies w = 11/16 = 0.6875
\]

Therefore, the portfolio weights would be as follows: 11/16 invested in the zero and 5/16 in the perpetuity.

b. Next year, the zero-coupon bond will have a duration of 4 years and the perpetuity will still have a 21-year duration. To obtain the target duration of nine years, which is now the duration of the obligation, we again solve for w:

\[
(w \times 4) + [(1 - w) \times 21] = 9 \implies w = 12/17 = 0.7059
\]

So, the proportion of the portfolio invested in the zero increases to 12/17 and the proportion invested in the perpetuity falls to 5/17.

18. a. From Rule 6, the duration of the annuity if it were to start in 1 year would be:

\[
\frac{1.10}{0.10} - \frac{10}{(1.10)^{10} - 1} = 4.7255 \text{ years}
\]

Because the payment stream starts in five years, instead of one year, we add four years to the duration, so the duration is 8.7255 years.
b. The present value of the deferred annuity is:

\[
\frac{10,000 \times \text{Annuity factor (10\%,10)}}{1.10^4} = 41,968
\]

Call \( w \) the weight of the portfolio invested in the 5-year zero. Then:

\[
(w \times 5) + [(1 – w) \times 20] = 8.7255 \Rightarrow w = 0.7516
\]

The investment in the 5-year zero is equal to:

\[
0.7516 \times 41,968 = 31,543
\]

The investment in the 20-year zeros is equal to:

\[
0.2484 \times 41,968 = 10,425
\]

These are the present or *market* values of each investment. The face values are equal to the respective future values of the investments. The face value of the 5-year zeros is:

\[
31,543 \times (1.10)^5 = 50,800
\]

Therefore, between 50 and 51 zero-coupon bonds, each of par value $1,000, would be purchased. Similarly, the face value of the 20-year zeros is:

\[
10,425 \times (1.10)^{20} = 70,134
\]

19. a. The Aa bond initially has a higher YTM (yield spread of 40 b.p. versus 31 b.p.), but it is expected to have a widening spread relative to Treasuries. This will reduce the rate of return. The Aaa spread is expected to be stable.

Calculate comparative returns as follows:

Incremental return over Treasuries =

\[
\text{Incremental yield spread} - (\text{Change in spread} \times \text{duration})
\]

Aaa bond: 31 bp – (0 \times 3.1 years) = 31 bp

Aa bond: 40 bp – (10 bp \times 3.1 years) = 9 bp

Therefore, choose the Aaa bond.
b. Other variables to be considered:

- Potential changes in issue-specific credit quality. If the credit quality of the bonds changes, spreads relative to Treasuries will also change.

- Changes in relative yield spreads for a given bond rating. If quality spreads in the general bond market change because of changes in required risk premiums, the yield spreads of the bonds will change even if there is no change in the assessment of the credit quality of these particular bonds.

- Maturity effect. As bonds near their maturity, the effect of credit quality on spreads can also change. This can affect bonds of different initial credit quality differently.

20. Using a financial calculator, we find that the actual price of the bond as a function of yield to maturity is:

<table>
<thead>
<tr>
<th>Yield to maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td>$1,620.45</td>
</tr>
<tr>
<td>8%</td>
<td>$1,450.31</td>
</tr>
<tr>
<td>9%</td>
<td>$1,308.21</td>
</tr>
</tbody>
</table>

Using the Duration Rule, assuming yield to maturity falls to 7%:

Predicted price change = \(\left(-\frac{\text{Duration}}{1+y}\right) \times \Delta y \times P_0\)

\[= \left(-\frac{11.54}{1.08}\right) \times (-0.01) \times $1,450.31 = $154.97\]

Therefore: predicted new price = $1,450.31 + $154.97 = $1,605.28

The actual price at a 7% yield to maturity is $1,620.45. Therefore:

% error = \(\frac{$1,605.28 – $1,620.45}{ $1,620.45}\) = 0.0094 = -0.94% (approximation is too low)

Using the Duration Rule, assuming yield to maturity increases to 9%:

Predicted price change = \(\left(-\frac{\text{Duration}}{1+y}\right) \times \Delta y \times P_0\)

\[= \left(-\frac{11.54}{1.08}\right) \times 0.01 \times $1,450.31 = -$154.97\]

Therefore: predicted new price = $1,450.31 – $154.97 = $1,295.34

The actual price at a 9% yield to maturity is $1,308.21. Therefore:

% error = \(\frac{$1,295.34 – $1,308.21}{ $1,308.21}\) = 0.0098 = -0.98% (approximation is too low)
Using Duration-with-Convexity Rule, assuming yield to maturity falls to 7%:

Predicted price change = \[
\left[ \left( -\frac{\text{Duration}}{1 + y} \right) \times \Delta y \right] + \left[ 0.5 \times \text{Convexity} \times (\Delta y)^2 \right] \times P_0
\]

\[= \left[ \left( -\frac{11.54}{1.08} \right) \times (-0.01) \right] + \left[ 0.5 \times 192.4 \times (-0.01)^2 \right] \times 1,450.31 = 168.92 \]

Therefore: predicted new price = $1,450.31 + $168.92 = $1,619.23
The actual price at a 7% yield to maturity is $1,620.45. Therefore:

\[\frac{1,620.45 - 1,619.23}{1,620.45} = 0.0075 = 0.075\% (\text{approximation is too low})\]

Using Duration-with-Convexity Rule, assuming yield to maturity rises to 9%:

Predicted price change = \[
\left[ \left( -\frac{\text{Duration}}{1 + y} \right) \times \Delta y \right] + \left[ 0.5 \times \text{Convexity} \times (\Delta y)^2 \right] \times P_0
\]

\[= \left[ \left( -\frac{11.54}{1.08} \right) \times 0.01 \right] + \left[ 0.5 \times 192.4 \times (0.01)^2 \right] \times 1,450.31 = -141.02 \]

Therefore: predicted new price = $1,450.31 – $141.02 = $1,309.29
The actual price at a 9% yield to maturity is $1,308.21. Therefore:

\[\frac{1,308.21 - 1,309.29}{1,308.21} = 0.0083 = 0.83\% (\text{approximation is too high})\]

Conclusion: The duration-with-convexity rule provides more accurate approximations to the true change in price. In this example, the percentage error using convexity with duration is less than one-tenth the error using only duration to estimate the price change.

21. a. The price of the zero coupon bond ($1,000 face value) selling at a yield to maturity of 8% is $374.84 and the price of the coupon bond is $774.84.

At a YTM of 9% the actual price of the zero coupon bond is $333.28 and the actual price of the coupon bond is $691.79.

Zero coupon bond:

Actual % loss = \[\frac{$333.28 - $374.84}{$374.84} = -0.1109 = 11.09\% \text{ loss}\]

The percentage loss predicted by the duration-with-convexity rule is:

Predicted % loss = \[\left( -11.81 \times 0.01 \right) + \left[ 0.5 \times 150.3 \times 0.01^2 \right] = -0.1106 = 11.06\% \text{ loss}\]
**Coupon bond:**

Actual % loss = \( \frac{\$691.79 - \$774.84}{\$774.84} = -0.1072 = 10.72\% \) loss

The percentage loss predicted by the duration-with-convexity rule is:

Predicted % loss = \( \left[ (-11.79) \times 0.01 \right] + \left[ 0.5 \times 231.2 \times 0.01^2 \right] = -0.1063 = 10.63\% \) loss

b. Now assume yield to maturity falls to 7%. The price of the zero increases to $422.04, and the price of the coupon bond increases to $875.91.

**Zero coupon bond:**

Actual % gain = \( \frac{\$422.04 - \$374.84}{\$374.84} = 0.1259 = 12.59\% \) gain

The percentage gain predicted by the duration-with-convexity rule is:

Predicted % gain = \( \left[ (-11.81) \times (-0.01) \right] + \left[ 0.5 \times 150.3 \times 0.01^2 \right] = 0.1256 = 12.56\% \) gain

**Coupon bond**

Actual % gain = \( \frac{\$875.91 - \$774.84}{\$774.84} = 0.1304 = 13.04\% \) gain

The percentage gain predicted by the duration-with-convexity rule is:

Predicted % gain = \( \left[ (-11.79) \times (-0.01) \right] + \left[ 0.5 \times 231.2 \times 0.01^2 \right] = 0.1295 = 12.95\% \) gain

c. The 6% coupon bond, which has higher convexity, outperforms the zero regardless of whether rates rise or fall. This can be seen to be a general property using the duration-with-convexity formula: the duration effects on the two bonds due to any change in rates are equal (since the respective durations are virtually equal), but the convexity effect, which is always positive, always favors the higher convexity bond. Thus, if the yields on the bonds change by equal amounts, as we assumed in this example, the higher convexity bond outperforms a lower convexity bond with the same duration and initial yield to maturity.

d. This situation cannot persist. No one would be willing to buy the lower convexity bond if it always underperforms the other bond. The price of the lower convexity bond will fall and its yield to maturity will rise. Thus, the lower convexity bond will sell at a higher initial yield to maturity. That higher yield is compensation for lower convexity. If rates change only slightly, the higher yield-lower convexity bond will perform better; if rates change by a substantial amount, the lower yield-higher convexity bond will perform better.
22. a. The following spreadsheet shows that the convexity of the bond is 64.933. The present value of each cash flow is obtained by discounting at 7%. (Since the bond has a 7% coupon and sells at par, its YTM is 7%.)

Convexity equals: the sum of the last column (7,434.175) divided by:

\[ P \times (1 + y)^2 = 100 \times (1.07)^2 \]

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Cash flow (CF)</th>
<th>PV(CF)</th>
<th>t + t²</th>
<th>(t + t²) x PV(CF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>6.542</td>
<td>2</td>
<td>13.084</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>6.114</td>
<td>6</td>
<td>36.684</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>5.714</td>
<td>12</td>
<td>68.569</td>
</tr>
<tr>
<td>4</td>
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Convexity: 64.933

The duration of the bond is (from rule 8):

\[ D = \left( \frac{1.07}{0.07} \right) \times \left( 1 - \frac{1}{1.07^{10}} \right) = 7.515 \text{ years} \]

b. If the yield to maturity increases to 8%, the bond price will fall to 93.29% of par value, a percentage decrease of 6.71%.

c. The duration rule predicts a percentage price change of:

\[ - \left( \frac{D}{1.07} \right) \times 0.01 = \left( \frac{-7.515}{1.07} \right) \times 0.01 = -0.0702 = -7.02\% \]

This overstates the actual percentage decrease in price by 0.31%.

The price predicted by the duration rule is 7.02% less than face value, or 92.98% of face value.

d. The duration-with-convexity rule would predict a percentage price change of

\[ - \left( \frac{-7.515}{1.07} \right) \times 0.01 + \left[ 0.5 \times 64.933 \times 0.01^2 \right] = -0.0670 = -6.70\% \]

The percentage error is 0.01%, which is substantially less than the error using the duration rule.

The price predicted by the duration rule is 6.70% less than face value, or 93.30% of face value.
23. a. \[ \% \text{ price change} = (-\text{Effective duration}) \times \text{Change in YTM} \% \]

CIC: \((-7.35) \times (-0.50\%) = 3.675\%\)

PTR: \((-5.40) \times (-0.50\%) = 2.700\%\)

b. Since we are asked to calculate horizon return over a period of only one coupon period, there is no reinvestment income.

\[ \text{Horizon return} = \frac{\text{Coupon payment} + \text{Year-end price} - \text{Initial Price}}{\text{Initial price}} \]

CIC: \[\frac{31.25 + 1,055.50 - 1,017.50}{1,017.50} = 0.06806 = 6.806\%\]

PTR: \[\frac{36.75 + 1,041.50 - 1,017.50}{1,017.50} = 0.05971 = 5.971\%\]

c. Notice that CIC is non-callable but PTR is callable. Therefore, CIC has positive convexity, while PTR has negative convexity. Thus, the convexity correction to the duration approximation will be positive for CIC and negative for PTR.

24. The economic climate is one of impending interest rate increases. Hence, we will seek to shorten portfolio duration.


b. The Arizona bond likely has lower duration. The Arizona coupons are slightly lower, but the Arizona yield is higher.

c. Choose the 12 3/8 \% coupon bond. The maturities are approximately equal, but the 12 3/8 \% coupon is much higher, resulting in a lower duration.

d. The duration of the Shell bond is lower if the effect of the higher yield to maturity and earlier start of sinking fund redemption dominates its slightly lower coupon rate.

e. The floating rate bond has a duration that approximates the adjustment period, which is only 6 months.
25. a. A manager who believes that the level of interest rates will change should engage in a rate anticipation swap, lengthening duration if rates are expected to fall, and shortening duration if rates are expected to rise.

b. A change in yield spreads across sectors would call for an intermarket spread swap, in which the manager buys bonds in the sector for which yields are expected to fall relative to other bonds and sells bonds in the sector for which yields are expected to rise relative to other bonds.

c. A belief that the yield spread on a particular instrument will change calls for a substitution swap in which that security is sold if its yield is expected to rise relative to the yield of other similar bonds, or is bought if its yield is expected to fall relative to the yield of other similar bonds.

26. While it is true that short-term rates are more volatile than long-term rates, the longer duration of the longer-term bonds makes their prices and their rates of return more volatile. The higher duration magnifies the sensitivity to interest-rate savings.

27. The minimum terminal value that the manager is willing to accept is determined by the requirement for a 3% annual return on the initial investment. Therefore, the floor is:

\[ \$1 \text{ million} \times (1.03)^5 = \$1.16 \text{ million} \]

Three years after the initial investment, only two years remain until the horizon date, and the interest rate has risen to 8%. Therefore, at this time, in order to be assured that the target value can be attained, the manager needs a portfolio worth:

\[ \frac{\$1.16 \text{ million}}{(1.08)^2} = \$0.9945 \text{ million} \]

This is the trigger point.

28. The maturity of the 30-year bond will fall to 25 years, and its yield is forecast to be 8%. Therefore, the price forecast for the bond is: \$893.25

[Using a financial calculator, enter the following: n = 25; i = 8; FV = 1000; PMT = 70] At a 6% interest rate, the five coupon payments will accumulate to \$394.60 after five years. Therefore, total proceeds will be: \$394.60 + \$893.25 = \$1,287.85
Therefore, the 5-year return is: \( \frac{\$1,287.85}{\$867.42} - 1 = 0.4847 \)
This is a 48.47% 5-year return, or 8.23% annually.

The maturity of the 20-year bond will fall to 15 years, and its yield is forecast to be 7.5%. Therefore, the price forecast for the bond is: \$911.73

[Using a financial calculator, enter the following: n = 15; i = 7.5; FV = 1000; PMT = 65] At a 6% interest rate, the five coupon payments will accumulate to \$366.41 after five years. Therefore, total proceeds will be: \$366.41 + \$911.73 = \$1,278.14
Therefore, the 5-year return is: \( \frac{\$1,278.14}{\$879.50} - 1 = 0.4533 \)
This is a 45.33% 5-year return, or 7.76% annually. The 30-year bond offers the higher expected return.
29. a. The advantages of bond indexing strategy are:
   - Historically, the majority of active managers underperform benchmark indexes in most periods; indexing reduces the possibility of underperformance at a given level of risk.
   - Indexed portfolios do not depend on advisor expectations or bets and so have less risk of underperforming the market.
   - Management advisory fees for indexed portfolios are dramatically smaller than fees for actively managed portfolios. Fees charged by active managers generally range from 15 to 50 basis points, while fees for indexed portfolios range from 1 to 20 basis points (with the highest of those representing enhanced indexing). Other non-advisory fees (i.e., custodial fees) are also smaller for indexed portfolios.
   - Plan sponsors have greater control over indexed portfolios because individual managers do not have as much freedom to vary from the parameters of the benchmark index. Some plan sponsors even decide to manage index portfolios with in-house investment staff.
   - Indexing is essentially “buying the market.” If markets are efficient, an indexing strategy should reduce unsystematic diversifiable risk, and should generate maximum return for a given level of risk.
   
   The disadvantages of bond indexing strategy are:
   - Indexed portfolio returns may match the bond index, but do not necessarily reflect optimal performance. In some time periods, many active managers may outperform an indexing strategy at the same level of risk.
   - The chosen bond index and portfolio returns may not meet the client objectives or the liability stream.
   - Bond indexing may restrict the fund from participating in sectors or other opportunities that could increase returns.

b. The stratified sampling, or cellular, method divides the index into cells, with each cell representing a different characteristic of the index. Common cells used in the cellular method combine (but are not limited to) duration, coupon, maturity, market sectors, credit rating, and call and sinking fund features. The index manager then selects one or more bond issues to represent the entire cell. The total market weight of issues held for each cell is based on the target index’s composition of that characteristic.

c. Tracking error is defined as the discrepancy between the performance of an indexed portfolio and the benchmark index. When the amount invested is relatively small and the number of cells to be replicated is large, a significant source of tracking error with the cellular method occurs because of the need to buy odd lots of issues to accurately represent the required cells. Odd lots generally must be purchased at higher prices than round lots. On the other hand, reducing the number of cells to limit the required number of odd lots would potentially increase tracking error because of the mismatch with the target.
30. a. A mismatch arises because Arseneault has more variable assets under management ($300 million) than variable liabilities ($200 million). The $300 million in assets has a shorter duration than an equivalent $300 million in liabilities that are split between $200 million variable rate and $100 million fixed rate. Consequently, Arseneault is exposed to interest rate risk. Should rates decline, the value of the liabilities will increase more rapidly than the value of the assets.

b Allied will receive 6.5% fixed rate from the investment bank and Allied will pay floating rate to the investment bank. The investment bank will pay fixed rate and receive floating rate. The notional amount will be $100 million, the difference between the $300 million in variable rate assets and the $200 million in variable rate liabilities. Allied pays LIBOR to Hogan Stanfield and receives 6.5% fixed from Hogan Stanfield. Allied pays 6.5% on $100 million in fixed rate bond liabilities and receives LIBOR from $100 million of variable rate assets.

31. a. If interest rates increase, Acree’s interest cost rises. Acree will pay a higher interest expense because the floating rate note resets its coupon every year based on 1-year LIBOR. The greater the increase in LIBOR, the more interest Acree will pay.

b. To hedge its interest rate exposure, Acree can enter into an interest rate swap agreement, in which the swap dealer pays LIBOR to Acree and Acree pays a fixed rate to the dealer. As LIBOR increases, the interest rate swap dealer pays the higher LIBOR to Acree, in exchange for Acree’s fixed rate payment to the swap dealer. The higher floating rate received by Acree in the swap offsets the higher interest expense on the floating rate note. Because Acree’s cost of funds is locked at the fixed rate, the increase in interest rates is absorbed by the swap dealer. The risk of increasing interest rates that Acree previously had is now shifted to the swap dealer, although Acree still bears the risk of decreasing interest rates.

32. \[ \Delta P/P = -D* \Delta y \]

For Strategy I:

5-year maturity: \[ \Delta P/P = -4.83 \times (-0.75\%) = 3.6225\% \]
25-year maturity: \[ \Delta P/P = -23.81 \times 0.50\% = -11.9050\% \]

Strategy I: \[ \Delta P/P = (0.5 \times 3.6225\%) + [0.5 \times (-11.9050\%)] = -4.1413\% \]

For Strategy II:

15-year maturity: \[ \Delta P/P = -14.35 \times 0.25\% = -3.5875\% \]

16-16
a. i. Strong economic recovery with rising inflation expectations. Interest rates and bond yields will most likely rise, and the prices of both bonds will fall. The probability that the callable bond will be called would decrease, and the callable bond will behave more like the non-callable bond. (Note that they have similar durations when priced to maturity). The slightly lower duration of the callable bond will result in somewhat better performance in the high interest rate scenario.

ii. Economic recession with reduced inflation expectations. Interest rates and bond yields will most likely fall. The callable bond is likely to be called. The relevant duration calculation for the callable bond is now modified duration to call. Price appreciation is limited as indicated by the lower duration. The non-callable bond, on the other hand, continues to have the same modified duration and hence has greater price appreciation.

b. Projected price change = (modified duration) × (change in YTM)

\[ = (-6.80) \times (-0.75\%) = 5.1\% \]

Therefore, the price will increase to approximately $105.10 from its current level of $100.

c. For Bond A, the callable bond, bond life and therefore bond cash flows are uncertain. If one ignores the call feature and analyzes the bond on a “to maturity” basis, all calculations for yield and duration are distorted. Durations are too long and yields are too high. On the other hand, if one treats the premium bond selling above the call price on a “to call” basis, the duration is unrealistically short and yields too low. The most effective approach is to use an option valuation approach. The callable bond can be decomposed into two separate securities: a non-callable bond and an option:

\[ \text{Price of callable bond} = \text{Price of non-callable bond} - \text{price of option} \]

Since the call option to call always has some positive value, the price of the callable bond is always less than the price of the non-callable security.
For the coupon bond, the weight on the last payment in the table above is less than it is in Spreadsheet 16.1 because the discount rate is higher; the weights for the first three payments are larger than those in Spreadsheet 16.1. Consequently, the duration of the bond falls. The zero coupon bond, by contrast, has a fixed weight of 1.0 for the single payment at maturity.

b. The weights for the 8% coupon bond are higher than in Spreadsheet 16.1, so duration decreases.
35.

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<th>Time (t)</th>
<th>Cash Flow</th>
<th>PV(CF)</th>
<th>( t + t^2 )</th>
<th>( (t + t^2) \times PV(CF) )</th>
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