13.1 EXPECTED RETURNS AND VARIANCES

A. Expected Return

Let $S$ denote the total number of states of the world, $R_i$ the return in state $i$, and $p_i$ the probability of state $i$. Then the expected return, $E(R)$, is given by:

$$E(R) = \sum_{i=1}^{S} (p_i \times R_i)$$

Example: High Tech

<table>
<thead>
<tr>
<th>State of economy</th>
<th>Probability of state</th>
<th>High Tech Return if state occurs</th>
<th>Collections Return if state occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>.25</td>
<td>-.05</td>
<td>.20</td>
</tr>
<tr>
<td>Average</td>
<td>.50</td>
<td>.15</td>
<td>0</td>
</tr>
<tr>
<td>Boom</td>
<td>.25</td>
<td>.35</td>
<td>-.15</td>
</tr>
</tbody>
</table>

1.00

Calculate the expected return for High Tech (Collection’s $E(R) = 1.25\%$).

B. Dispersion

$$\text{Var}(R) = \sigma^2 = \sum_{i=1}^{S} \left[ p_i \times (R_i - E(R))^2 \right]$$

$$\sigma = \sqrt{\text{Var}(R)}$$

$$CV = \frac{\sigma}{E(R)}$$

Calculate the variance, standard deviation, of coefficient of variation of returns for High Tech. Collections: $\text{Var}(R) = 0.0155; \ \sigma = 0.1244; \ \text{CV} = 9.95$

13.2 PORTFOLIOS

A portfolio is a collection of securities, such as stocks and bonds, held by an investor.

A. Portfolio Weights

Portfolios can be described by the percentages of the portfolio's total value invested in each security, i.e., by the portfolio weights.

Example:

If two securities in a portfolio have a combined value of $10,000 and $6,000 is invested in High Tech and $4,000 in Collections, then what are the portfolio weights of High Tech and Collections?
B. Portfolio Expected Returns
Example:
If the expected return on High Tech stock is 15% and that of Collections is 1.25%, and $6,000 is invested in High Tech while $4,000 is invested in Collections, calculate the portfolio expected return.
\[ E(R_p) = \sum x_i E(R_i) \]

C. Portfolio Standard Deviation
Unlike expected return, the standard deviation of a portfolio is not the weighted sum of the individual security variances. Combining securities into portfolios can reduce the total variability of returns.
Important concept: Correlation of returns.
Example:
Consider the above portfolio with 60% in High Tech and 40% in Collections:

<table>
<thead>
<tr>
<th>State of economy</th>
<th>Probability of state</th>
<th>Return on High Tech</th>
<th>Return on Collections</th>
<th>Return on portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>.25</td>
<td>-.05</td>
<td>.20</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>.50</td>
<td>.15</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Boom</td>
<td>.25</td>
<td>.35</td>
<td>-.15</td>
<td></td>
</tr>
<tr>
<td>Expected return</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate the expected return and standard deviation of the portfolio.

13.4 RISK: SYSTEMATIC AND UNSYSTEMATIC
A. Systematic and Unsystematic Risk
Systematic risk—a surprise that affects a large number of assets, each to a greater or lesser extent—sometimes called market risk.

Unsystematic risk—a risk or surprise that affects at most a small number of assets—sometimes called unique risk.

B. Systematic and Unsystematic Components of Return
Total return = Expected return + Unexpected return

\[ R = E(R) + U \]
\[ R = E(R) + \text{Systematic portion} + \text{Unsystematic portion} \]
\[ R = E(R) + m + \varepsilon \]
Where \( m \) denotes the market or systematic portion and \( \varepsilon \) represents the unique or unsystematic portion of risk.
13.5 DIVERSIFICATION AND PORTFOLIO RISK

A. The Principle of Diversification (see figure 13.1 on page 428 in text)

Principle of diversification—principle stating that combining imperfectly correlated assets can produce a portfolio with less variability than the typical individual asset.

The portion of variability present in a typical single security that is not present in a portfolio of securities is termed diversifiable risk. The level of variance that is present in portfolios of assets is termed undiversifiable risk.

B. Diversification and Unsystematic Risk

When securities are combined into portfolios, their unique or unsystematic risks tend to cancel each other out, leaving only the variability that affects all securities to a greater or lesser degree. Thus, diversifiable risk is synonymous with unsystematic risk. Large portfolios have little or no unsystematic risk.

C. Diversification and Systematic Risk

Systematic risk cannot be eliminated by diversification since it represents the variability due to influences that affect all securities to a greater or lesser extent. Thus, systematic risk and undiversifiable risk are analogous.

13.6 SYSTEMATIC RISK AND BETA

A. The Systematic Risk Principle

The reward for bearing risk depends only upon the systematic or undiversifiable risk of an investment (since unsystematic risk can be diversified away).

The implication: The expected return on an asset depends only upon that asset's systematic risk.

B. Measuring Systematic Risk

Beta coefficient (β)—measure of how much systematic risk an asset has relative to an average risk asset.

C. Portfolio Betas

While portfolio variance is not equal to a simple weighted sum of individual security variances, a portfolio beta is equal to the weighted sum of individual security betas. \( \beta_p = \sum x_i \beta_i \)

Example:
Calculate the beta of the following portfolio:

<table>
<thead>
<tr>
<th>Stock</th>
<th>Amount invested</th>
<th>Portfolio weight</th>
<th>Beta coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Tech</td>
<td>$6,000</td>
<td>60%</td>
<td>1.29</td>
</tr>
<tr>
<td>Collections</td>
<td>$4,000</td>
<td>40%</td>
<td>-0.86</td>
</tr>
<tr>
<td>Portfolio</td>
<td>$10,000</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>
13.7 THE SECURITY MARKET LINE

Capital Asset Pricing Model (CAPM): A model based on the proposition that any stock's required rate of return is equal to the risk-free rate of return plus a risk premium.

The CAPM states that the expected return on asset depends upon:
1. The time value of money, as measured by $R_f$.
2. The reward per unit of systematic risk, $E(R_m) - R_f$.
3. The asset's systematic risk, as measured by $\beta_i$.

To invest in the market portfolio, $m$, you will require a rate of return, $k_m$, greater than the risk-free rate, $k_{rf}$.

$$E(R_m) = R_f + \text{market risk premium}$$
$$E(R_m) = R_f + (R_m - R_f)$$

The market risk premium is the additional return over the risk-free rate needed to compensate investors for assuming the average amount of risk, $(R_m - R_f)$.

To invest in a risky security, $j$, you will require a rate of return greater than the risk-free rate. The risk premium for security $j$ can be stated relative to the market risk premium, $(R_m - R_f)\beta_j$.

$$R_j = R_f + \text{risk premium}$$
$$R_j = R_f + (R_m - R_f)\beta_j$$

Example:
Assume Treasury bonds yield = 8%, the average stock's required return = 15%, and the beta for security High Tech is 1.29. What is the market risk premium?
What is the risk premium for High Tech?
What is the required rate of return for High Tech?

Security Market Line (SML) is the line that shows the relationship between risk as measured by beta and the required rate of return for individual securities.